1.1 Limits Of A Function

Standards:

MCA1

MCA1b

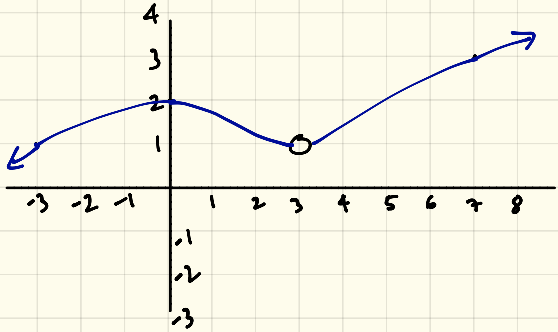
MCA2

MCA2a

MCA2b

Old: Evaluating Functions

Let's consider the following graph:

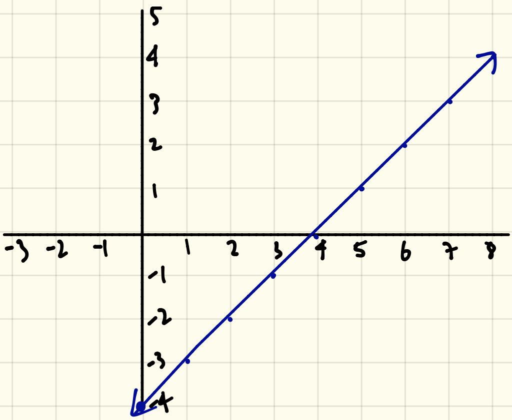


Answer the following:

1. $f(0) = 2$
2. $f(3) = \text{undefined}$
3. $f(-3) = 1$
4. $f(7) = 3$

New: Evaluating Limits of a Function

Let's consider the function: $y = x - 4$. Let's say that we take numbers close to 5 (4.9, 4.99, 4.999, 5.001, 5.01, 5.1) and we substitute those numbers into the function. When we do this what happens?



x	y
4.9	.9
4.99	.99
4.999	.999
5.001	1.001
5.01	1.01
5.1	1.1

5

1

As the x-values gets closer to 5, it seems the y-values get closer to 1.

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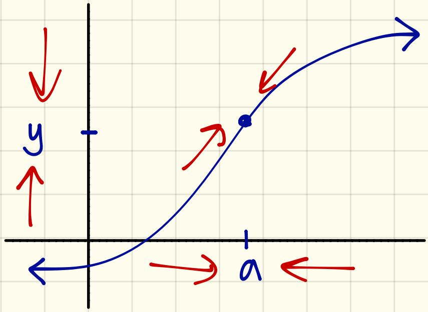
Basic Idea of a Limit

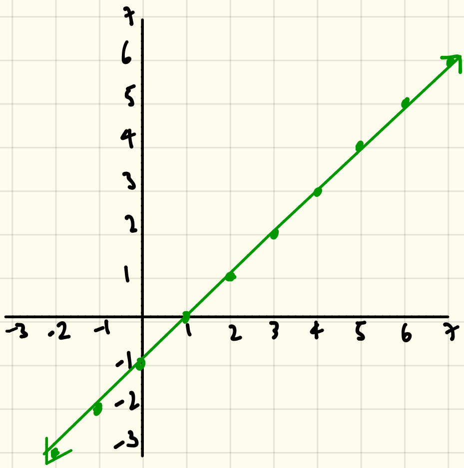
It's where the y-values "tend to go" as the x-values approach a certain number

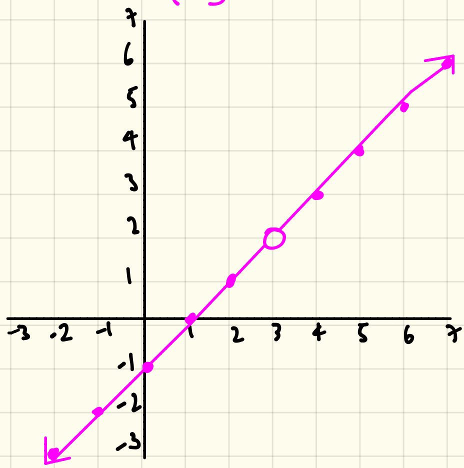
$$\lim_{x \to a} f(x) = y$$



Let's consider the graph's of these 2 functions:

$f(x) = x - 1$ and $g(x) = \dfrac{x^2 - 4x + 3}{x - 3}$





$$\boxed{\begin{array}{l} f(3) = 2 \\ \lim_{x \to 3} f(x) = 2 \end{array}}$$

x	y
2.9	1.9
2.99	1.99
2.999	1.999
3.001	2.001
3.01	2.01
3.1	2.1

$$g(x) = \frac{x^2 - 4x + 3}{x - 3} = \frac{(x-1)\cancel{(x-3)}}{\cancel{(x-3)}} = x - 1$$

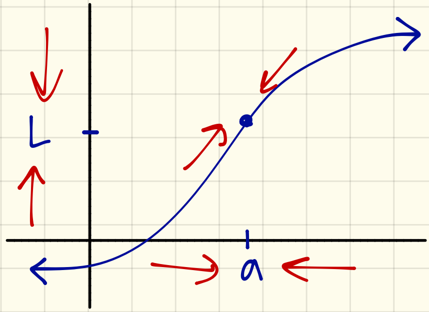
$g(x)$ acts like $x - 1$ but $x \neq 3$.

$$\boxed{\begin{array}{l} g(3) = \text{undefined} \\ \lim_{x \to 3} g(x) = 2 \end{array}}$$

Conclusion Actual values of the function does not matter. It's where the y-values approach that matter for limits.

Definition of a Limit

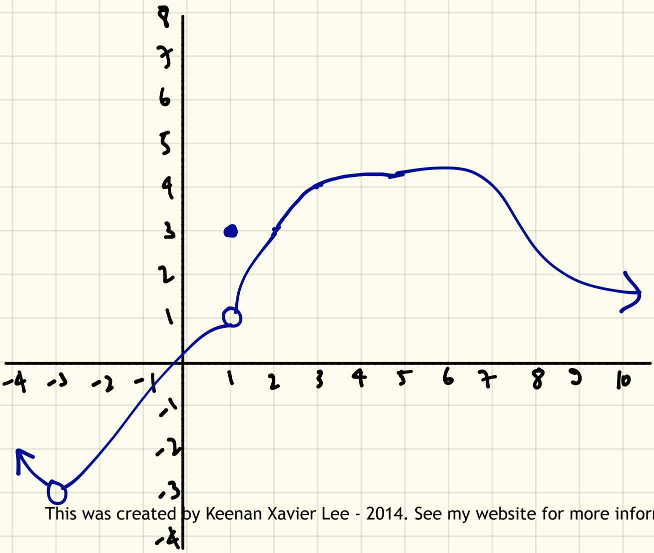
$$\lim_{x \to a} f(x) = L$$



This means ...
"the limit of $f(x)$, as x approaches (a), equals L."

If we take the values of $f(x)$ arbitrarily close to L (as close to L as we want) by taking x to be sufficiently close to a (on either side) but not equal to a.

[Example 1] Evaluate.



a) $f(1) = 3$

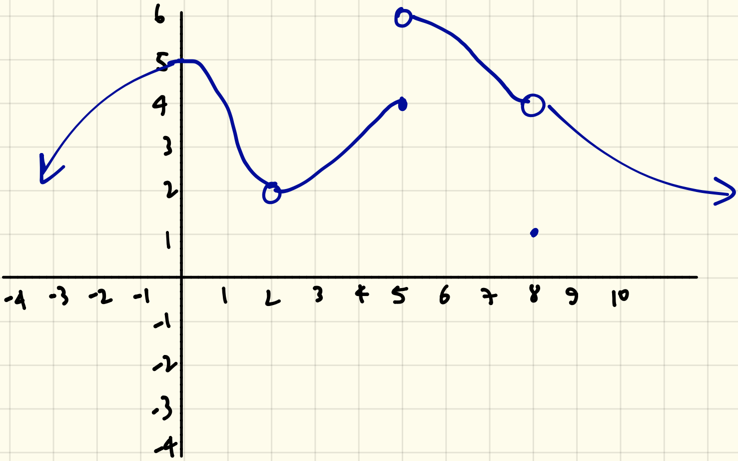
b) $\lim_{x \to 1} f(x) = 1$

c) $f(3) = 4$

d) $f(2) = 3$

e) $\lim_{x \to 3} f(x) = 4$

[Example 2] Evaluate.



a) $f(2) =$ undefined

b) $f(5) = 4$

c) $\lim_{x \to 5} f(x) =$ D.N.E

d) $f(8) = 1$

e) $\lim_{x \to 8} f(x) = 4$

f) $f(0) = 5$

g) $\lim_{x \to 0} f(x) = 5$

One-sided Limits

One-sided limits are almost the same concept as "two-sided limits" but we will be taking the limits from ONLY one side of the particular x-value.

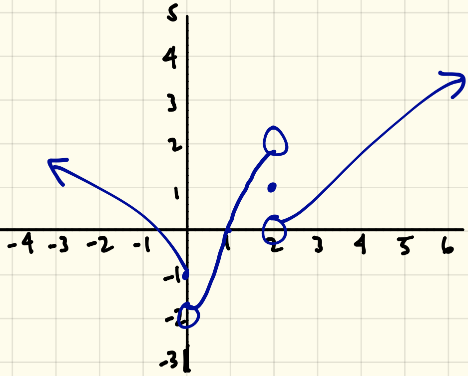
1. $\lim_{x \to a^-} f(x) =$ limit as x approaches (a) from the left side (for values less than a).

2. $\lim_{x \to a^+} f(x) =$ Limit as x approaches (a) from the right side (for values greater than a)

FACT

$\lim_{x \to a} f(x) = L$ if and only if $\lim_{x \to a^-} f(x) = L$ and $\lim_{x \to a^+} f(x) = L$

[Example 3] Evaluate.



a) $\lim_{t \to 0^-} g(t) = -1$

b) $\lim_{t \to 0^+} g(t) = 2$

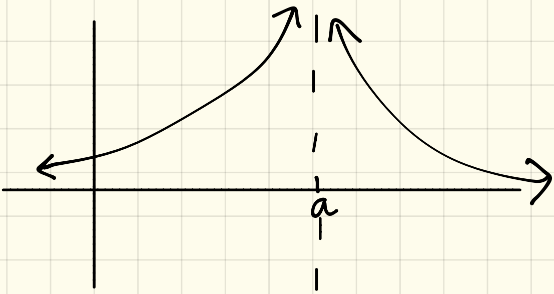
c) $\lim_{t \to 0} g(t) =$ D.N.E

d) $\lim_{t \to 2^-} g(t) = 2$

e) $\lim_{t \to 2} g(t) =$ D.N.E

f) $g(2) = 1$

Infinite Limits



Sometimes functions contain VERTICAL ASYMPTOTES, where x approaches (a), the y-values get larger & larger. Since (a) does not exist, y-values will get really close to a defined value but will NEVER become defined. So we say the y-values "take off" to infinity.

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Definition of Infinite Limits

$$\lim_{x \to a} f(x) = \infty \quad \text{or} \quad \lim_{x \to a} f(x) = -\infty$$

"f(x) becomes infinite as x approaches a".

- If x approaches (a) & y-values get positively larger & larger, we say it goes to infinity (∞)
- If x approaches (a) & y-values get negatively larger & larger, we say it goes to infinity ($-\infty$)

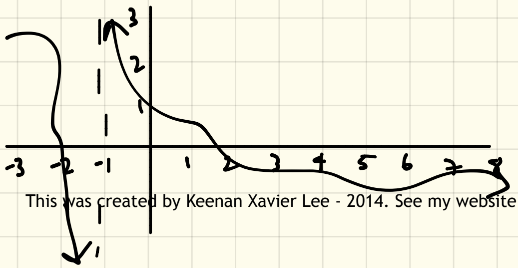
Vertical Asymptotes

If at least 1 of the following is true, then there exist a vertical asymptote:

$\boxed{1}$ $\lim_{x \to a} f(x) = \infty$ $\boxed{2}$ $\lim_{x \to a^-} f(x) = \infty$ $\boxed{3}$ $\lim_{x \to a^+} f(x) = \infty$

$\boxed{4}$ $\lim_{x \to a} f(x) = -\infty$ $\boxed{5}$ $\lim_{x \to a^-} f(x) = -\infty$ $\boxed{6}$ $\lim_{x \to a^+} f(x) = -\infty$

[Example 4] Evaluate.

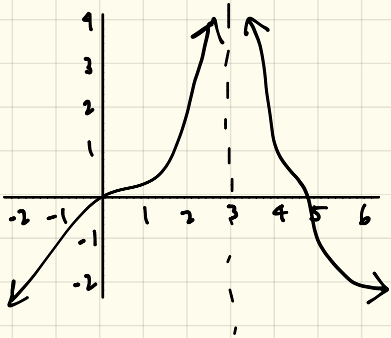


$\boxed{a}$ $\lim_{x \to -1^-} f(x) = -\infty$

$\boxed{b}$ $\lim_{x \to -1^+} f(x) = \infty$

$\boxed{c}$ $\lim_{x \to 1} f(x) =$ D.N.E

[Example 5] Evaluate.



a) $\lim_{x \to 3^-} f(x) = \infty$

b) $\lim_{x \to 3^+} f(x) = \infty$

c) $\lim_{x \to 3} f(x) = \infty$

[Example 6] Evaluate.

a) $\lim_{x \to -5^-} f(x) = \infty$

b) $\lim_{x \to 5} f(x) =$ D.N.E

c) $\lim_{x \to 0} f(x) =$ D.N.E

d) $\lim_{x \to 2} f(x) = -2.5$

e) $\lim_{x \to -1} f(x) = 3$

[Example 7] Sketch the graph of an example of a function satisfying the following conditions:

$\lim_{x \to 1} f(x) = \infty$, $\lim_{x \to 4} f(x) = 3$, $f(-2) = 0$