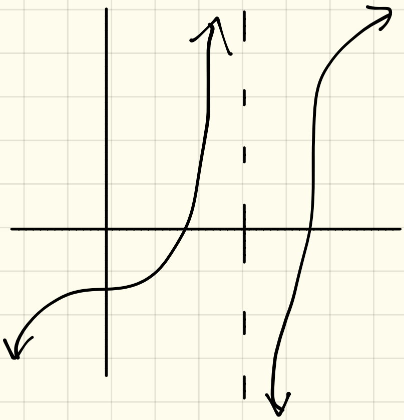
# 1.3 Limits Involving Infinity

Standards:

MCA2

MCA2c

## Old Infinite Limits



a $\lim_{x \to 3^-} f(x) = \infty$

b $\lim_{x \to 3^+} f(x) = -\infty$

c $\lim_{x \to 3} f(x) = $ D.N.E

Infinite Limits involve VERTICAL ASYMPTOTES.

## new Limits involving infinity

Now we are learning how to evaluate limits of functions as x approaches infinity ($\infty$ or $-\infty$) graphically & algebraically.

What we know:

$\lim_{x \to a} f(x) = L$

What we are learning now:

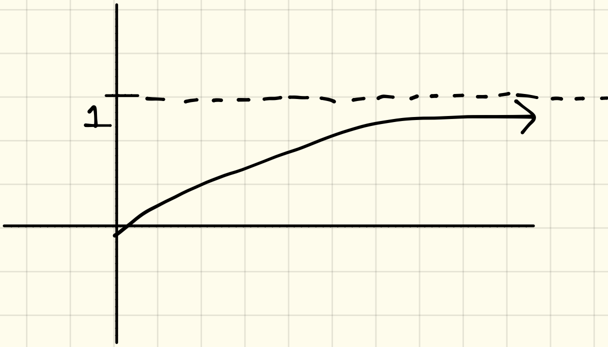
$\lim_{x \to \infty} f(x) = L$

or

$\lim_{x \to -\infty} f(x) = L$

Let's consider the function: $f(x) = \frac{x^2 - 1}{x^2 + 1}$

| x | y |
|---|---|
| 0 | -1 |
| 1 | 0 |
| 2 | .6 |
| 3 | .8 |
| ... | ... |
| 6 | .94 |
| ... | ... |
| 20 | .99 |

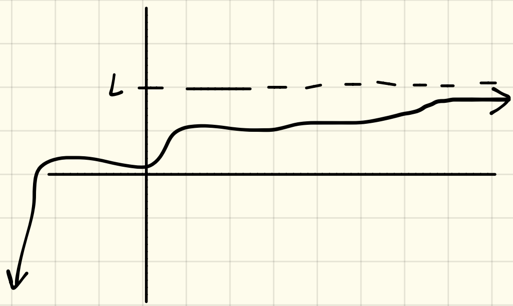



Conclusion $\lim_{x \to \infty} \frac{x^2 - 1}{x^2 + 1} = .\overline{99} = 1$

As the x-values go towards infinity, the y-values "tend to go" & "settle down" at 1.

Basic Idea Limits involving infinity is analyzing the end behavior of a function where the y-value "settle down" to a specific number. This specific number is called the <u>HORIZONTAL</u> <u>ASYMPTOTE</u>.

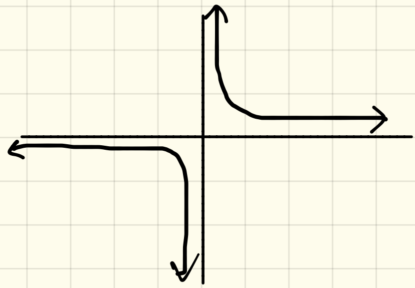
$\lim_{x \to \infty} f(x) = L$




This was created by Keenan Xavier Lee - 2014. See my website for more information, lee-apcalculus.weebly.com.

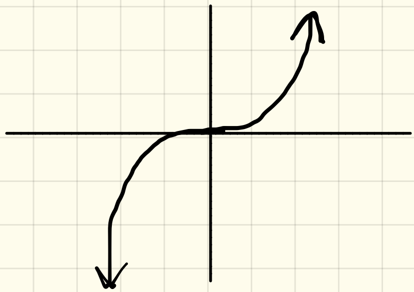
[Examples] ① $\lim_{x\to\infty} \frac{1}{x}$

② $\lim_{x\to-\infty} \frac{1}{x}$



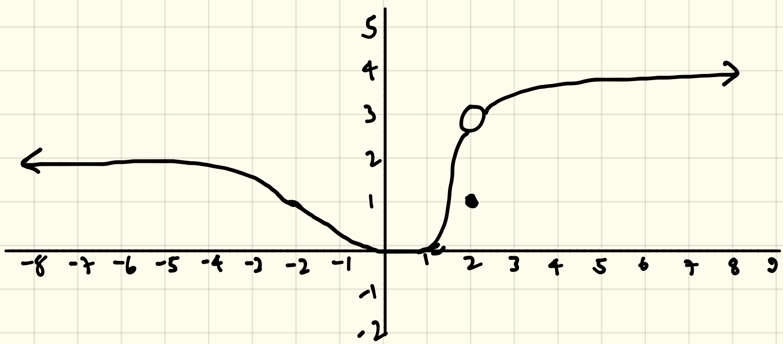
① $\lim_{x\to\infty} \frac{1}{x} = 0$  ② $\lim_{x\to-\infty} \frac{1}{x} = 0$

[Example 3] $\lim_{x\to\infty} x^3$



$\lim_{x\to\infty} x^3 = $ D.N.E

[Example 4] Evaluate.




① $\lim_{x\to 2} f(x) = 3$  ② $\lim_{x\to 2^-} f(x) = 3$  ③ $\lim_{x\to\infty} f(x) = 4$  ④ $\lim_{x\to 0} f(x) = 0$

⑤ $f(2) = 1$  ⑥ $\lim_{x\to-\infty} f(x) = 2$  ⑦ $\lim_{x\to-2^-} f(x) = 1$

# Calculating Limits Algebraically

To evaluate limits of functions as x approaches $\infty$ (or $-\infty$), the manipulation technique to use is to divide the numerator & denominator by the highest degree of x that occurs in the denominator.

[Example 5] $\lim\limits_{x\to\infty} \dfrac{3x^3-7x^2+2x-6}{4x^3-13x-1}$

$$\lim_{x\to\infty} \frac{3x^3-7x^2+2x-6}{4x^3-13x-1}\cdot\frac{\frac{1}{x^3}}{\frac{1}{x^3}} = \lim_{x\to\infty}\frac{\frac{3x^3}{x^3}-\frac{7x^2}{x^3}+\frac{2x}{x^3}-\frac{6}{x^3}}{\frac{4x^3}{x^3}-\frac{13x}{x^3}-\frac{1}{x^3}}$$

$$= \lim_{x\to\infty}\frac{3-\overbrace{\frac{7}{x}}^{\to 0}+\overbrace{\frac{2}{x^2}}^{\to 0}-\overbrace{\frac{6}{x^3}}^{\to 0}}{4-\underbrace{\frac{13}{x^2}}_{\to 0}-\underbrace{\frac{1}{x^3}}_{\to 0}} = \frac{3}{4}.$$

[Example 6] $\lim\limits_{x\to\infty} \dfrac{2x^2+x-1}{x^2+x-2}$

$$\lim_{x\to\infty}\frac{2x^2+x-1}{x^2+x-2}\cdot\frac{\frac{1}{x^2}}{\frac{1}{x^2}} = \lim_{x\to\infty}\frac{\frac{2x^2}{x^2}+\frac{x}{x^2}-\frac{1}{x^2}}{\frac{x^2}{x^2}+\frac{x}{x^2}-\frac{2}{x^2}} = \lim_{x\to\infty}\frac{2+\overbrace{\frac{1}{x}}^{\to 0}-\overbrace{\frac{1}{x^2}}^{\to 0}}{1+\underbrace{\frac{1}{x}}_{\to 0}-\underbrace{\frac{2}{x^2}}_{\to 0}}$$

$$= 2.$$

[Example 7] $\lim\limits_{u\to\infty} \dfrac{4u^4+5}{(u^2-2)(2u^2-1)}$

$$\lim_{u\to\infty} \frac{4u^4+5}{(u^2-2)(2u^2-1)} = \lim_{u\to\infty} \frac{4u^4+5}{2u^4-u^2-4u^2+2} = \lim_{u\to\infty} \frac{4u^4+5}{2u^4-5u^2+2} \cdot \frac{\frac{1}{u^4}}{\frac{1}{u^4}}$$

$$= \lim_{u\to\infty} \frac{\frac{4u^4}{u^4}+\frac{5}{u^4}}{\frac{2u^4}{u^4}-\frac{5u^2}{u^4}+\frac{2}{u^4}} = \lim_{u\to\infty} \frac{4+\underbrace{\frac{5}{u^4}}_{\to 0}}{2-\underbrace{\frac{5}{u^2}}_{\to 0}+\underbrace{\frac{2}{u^4}}_{\to 0}} = 2.$$