1.4 Continuity

Standards:

MCA2

MCA2d

$\boxed{\text{Old}}$ Direct Substitution Property

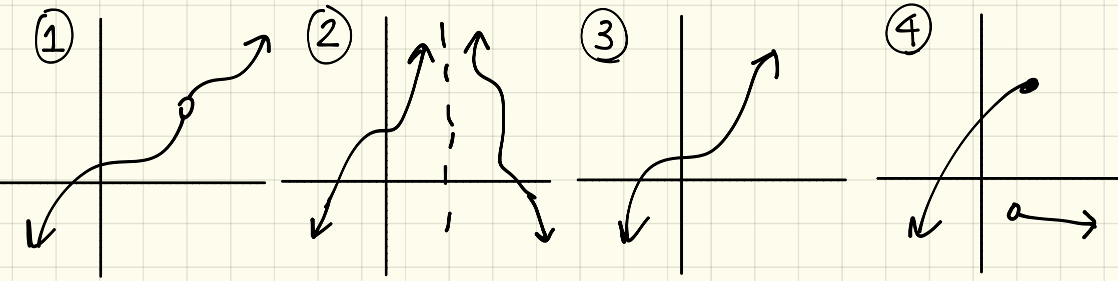
If function f is a polynomial/rational & (a) is in the domain, then

$$\lim_{x \to a} f(x) = f(a).$$

$\boxed{\text{FACT}}$ Functions that applied this property are called continuous at (a).

$\boxed{\text{new}}$ Continuity

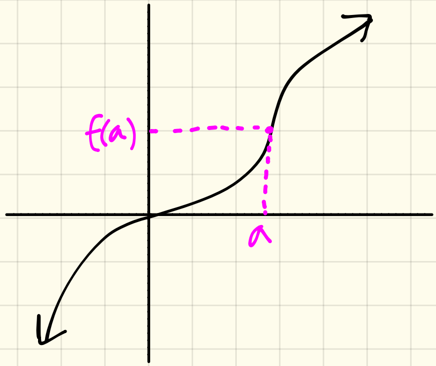
Which graph(s) do you think is/are continuous?



Graph #3 is the only one that is continuous. The basic idea for picking a graph to be continuous is drawing the graph without "picking up your pencil".

Calculus Definition of Continuity

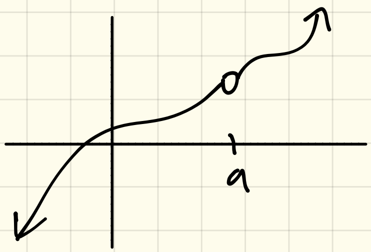
A function f is continuous at point (a) if $\lim_{x \to a} f(x) = f(a)$.



Basically this definition is saying that f is continuous at (a) if $f(x)$ approaches $f(a)$ as x approaches (a). It is every number in an interval of f whose graph has <u>no</u> break.

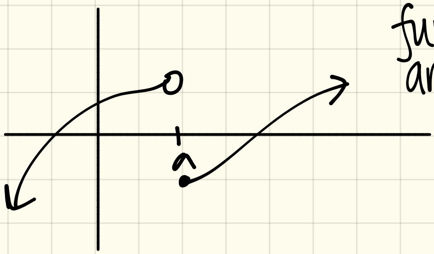
3 types of Discontinuities

1 Removeable Discontinuity



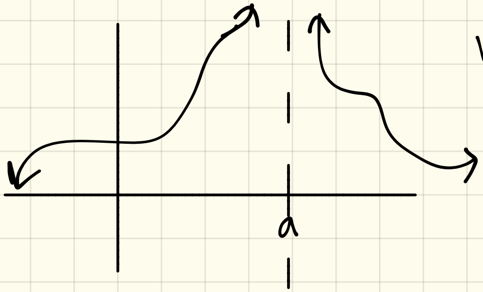
Value is <u>NOT</u> in the domain of the function — the value has been "removed" from the domain.

2 Jump Discontinuity



function "jumps" from one value to another

3 Infinite Discontinuity



vertical asymptote is present at (a).

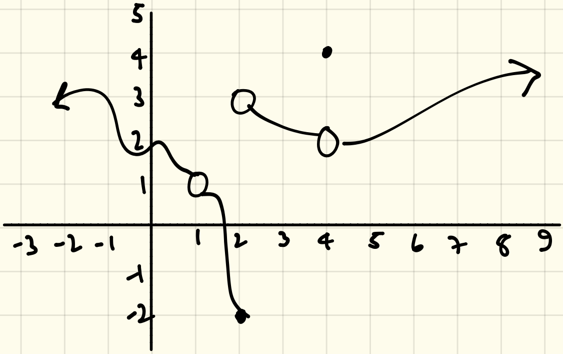
3 Requirements for a function to be continuous

Condition 1: $f(a)$ is defined

Condition 2: $\lim_{x \to a} f(x)$ must exist

Condition 3: $\lim_{x \to a} f(x) = f(a)$.

[Example 1] Let's consider a point that is continuous & use the requirements to prove that is continuous.



Let's look at $x = 6$.

$x = 6$

✓ Condition 1: $f(6)$ is defined

✓ Condition 2: $\lim_{x \to 6} f(x)$ exist

✓ Condition 3: $\lim_{x \to 6} f(x) = f(6)$

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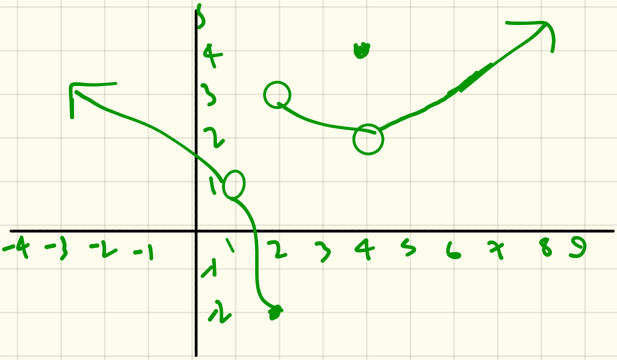


$x = 6$

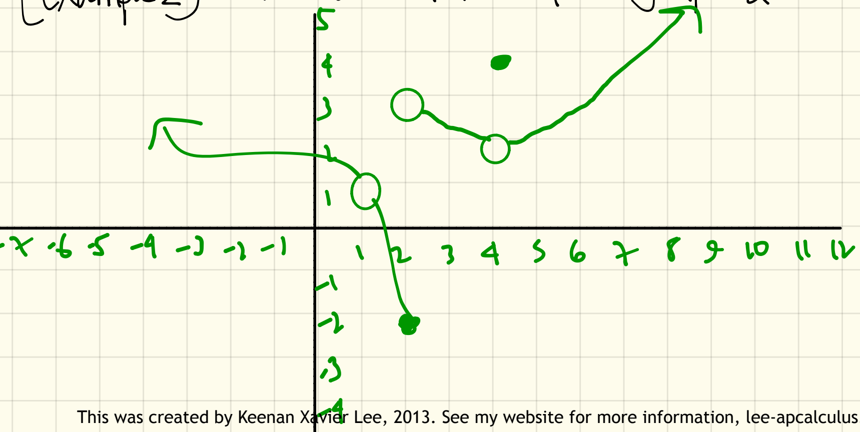
✓ condition 1: $f(6) = 3$

✓ condition 2: $\lim_{x \to 6} f(x) = 3$

✓ condition 3: $\lim_{x \to 6} f(x) = f(6)$

$3 = 3$.

[Example 2] Find the discontinuous of the graph & identify reason why.



$x=1$

X condition 1: $f(1) = $ d.n.e

$x=2$

✓ condition 1: $f(2) = 2$

X condition 2: $\lim\limits_{x\to 2} f(x) = $ d.n.e.

$x=4$

condition 1: $f(4) = 4$

condition 2: $\lim\limits_{x\to 4} f(x) = 2$

X condition 3: $\lim\limits_{x\to 4} f(x) = f(4)$

$2 \neq 4$

[Examples] Where are each of these functions discontinuous. Give a reason why.

② $f(x) = \dfrac{x^2 - x - 2}{x - 2}$ — discontinuous at $x=2$

X condition 1: $f(2) = $ d.n.e

③ $f(x) = \begin{cases} \dfrac{x^2 - x - 2}{x - 2} & \text{if } x \neq 2 \\ 1 & \text{if } x = 2 \end{cases}$ — discontinuous at $x=2$

✓ condition 1: $f(2) = 1$

✓ condition 2: $\lim\limits_{x\to 2} \dfrac{x^2 - x - 2}{x - 2} = \lim\limits_{x\to 2} \dfrac{\cancel{(x-2)}(x+1)}{\cancel{(x-2)}}$

$= \lim\limits_{x\to 2} x+1 = (2)+1 = 3$

X condition 3: $\lim\limits_{x\to 2} \dfrac{x^2 - x - 2}{x - 2} \neq f(2)$

$3 \neq 1$

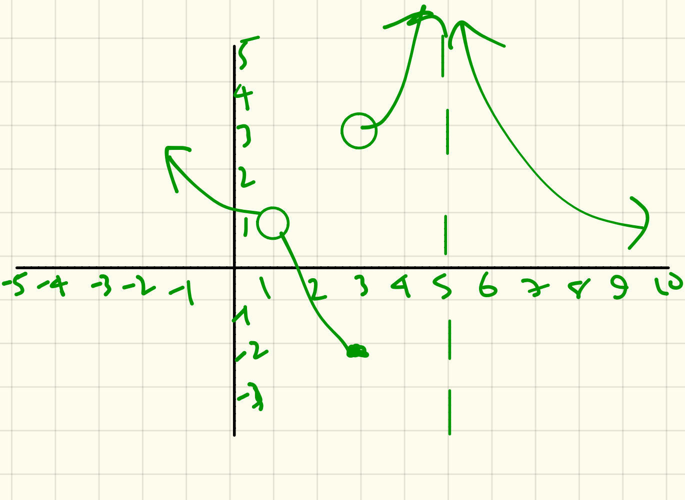
One-sided Continuity - The basic idea is the same except we are looking for continuity of one side of the value, rather than both sides.

Definition:

- A function f is continuous from the right if $\lim_{x \to a^+} f(x) = f(a)$.

- A function is continuous from the left if $\lim_{x \to a^+} f(x) = f(a)$.

[Example 4]



ⓐ Is $x=1$ continuous from the left? no

$$\lim_{x\to1^-} f(x) = f(1)$$
$$1 \neq \text{d.n.e}$$

Is $x=1$ continuous from the right? no

$$\lim_{x\to1^+} f(x) = f(1)$$
$$1 \neq \text{d.n.e}$$

ⓑ Is $x=3$ continuous from the left? yes

$$\lim_{x\to3^-} f(x) = f(3)$$
$$-2 = -2$$

Is $x=3$ continuous from the right? no

$$\lim_{x\to3^+} f(x) = f(3)$$
$$3 \neq -2$$

[Example 5] Sketch the graph of the function with the following conditions.

- $\lim_{x\to3^-} f(x) = \infty$
- $\lim_{x\to5^+} f(x) = \infty$
- $f(1) = 2$
- $\lim_{x\to1} f(x) = 3$.