1.6 Estimating Tangent Lines

Standards:

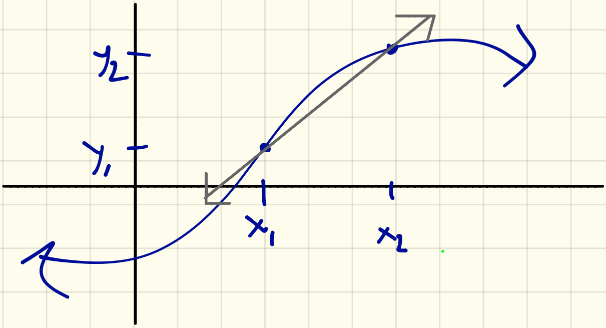
MCD1

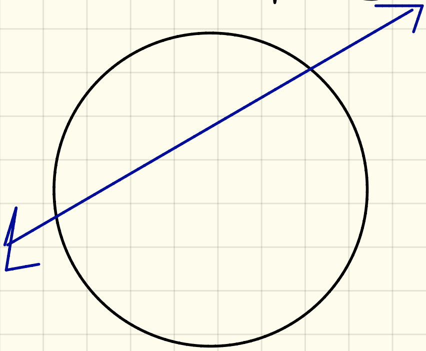
MCD1a

MCD1b

$\boxed{\text{Old}}$ Secant Lines

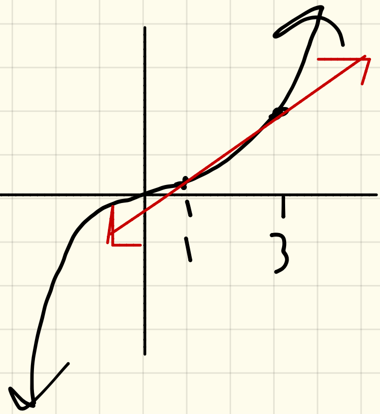
What is a secant line? A secant line is is a line passing through 2 points on a curve.





secant lines give average rate of change.

[Example] Find the slope of the secant line of $y = x^3$ on the interval $[1,3]$.



$$\text{Slope} = m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$= \frac{27 - 1}{3 - 1} = 13$$

What is the equation of the secant line?

$y = mx + b \Rightarrow$ What do know?

$m = 13, \; x = 1, \; y = 1.$

$1 = 13(1) + b$

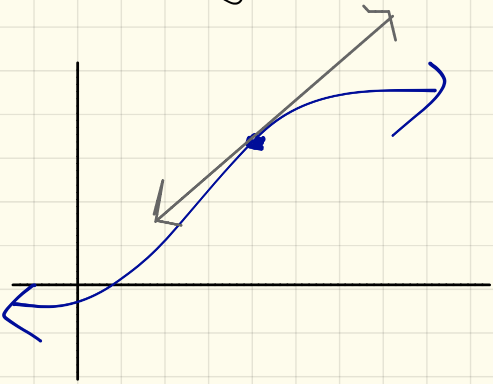
$1 = 13 + b$

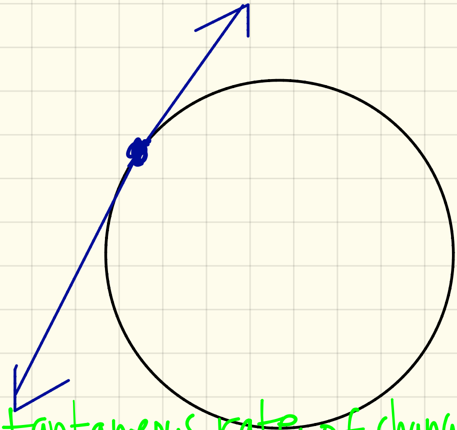
$-12 = b.$

$\boxed{y = 13x - 12}$

$\boxed{\text{new}}$ Tangent Lines

What is a tangent line? A tangent line is a line that touches the graph in <u>one</u> local point.





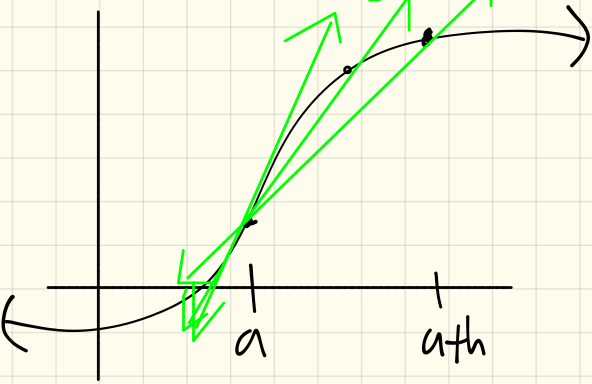
tangent lines give <u>instantaneous</u> rate of change.

Tangent Lines

- Tangent lines are instantaneous rate of change.
- It's a better way to study the exact behavior of graphs at every little section & point.
- Think of it as an easy "snapshot" of what is going on in the function.

Relationship between Secant & Tangent Lines

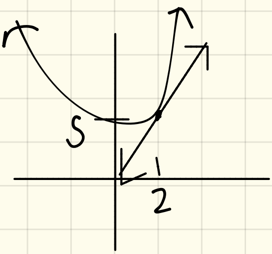
As the secant lines approaches the tangent lines, the secant lines eventually converges into the tangent line



The gradual jump from average to instantaneous requires the limit notion.

Tangent line at x = a. $\frac{f(a+h)-f(a)}{h} \Rightarrow \lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$

[Example 1] Let's consider the graph of $y=x^2+5$. Find the instantaneous rate of change at $x=2$.



X	$\frac{f(2)-f(x)}{2-x}$
1.9	3.9
1.99	3.99
1.999	3.999
2.001	4.001
2.01	4.01
2.1	4.1

2 → 4

At x=2, the slope of the tangent line is 4.

[Example 2] Estimate the slope at point (1,2) to $f(x) = 2x^3 - x + 1$.

[Example 3] A stone is released from a state of rest falling to Earth. Let the function describing the situation be $f(t) = 16t^2$. Estimate the slope of the tangent line at $t = 0.5$ seconds.