

1.7 Calculating Tangent Lines

Standards:

MCD1

MCD1a

MCD1b

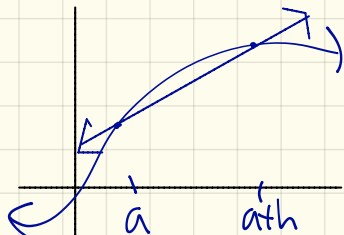
MCD1c



old → We "guessed" or estimated values of slopes of tangent lines on the basis of numerical evidence

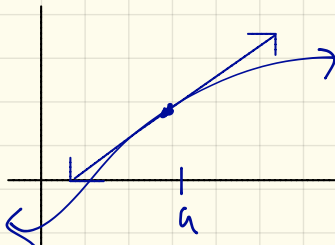
new → we are going to algebraically evaluate the values of the tangent lines

Let's remember the following formulas:



Slope of secant line

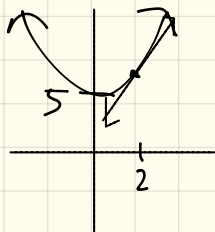
$$m = \frac{f(a+h) - f(a)}{h}$$



Slope of tangent line

$$m = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

[Example 1a] Estimate the slope of the tangent line for $f(x) = x^2 + 5$ when $x = 2$



x	$\frac{f(2) - f(x)}{2 - x}$
1.9	3.9
1.99	3.99
1.999	3.999
2.001	4.001
2.01	4.01
2.1	4.1

Arrows indicate the values approaching 4 as x approaches 2.

(Example 1b) Evaluate the slope of the tangent line for $f(x) = x^2 + 5$ when $x=2$.

$$m = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{f(2+h) - f(2)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{[(2+h)^2 + 5] - [9]}{h}$$

$$= \lim_{h \rightarrow 0} \frac{[4 + 4h + h^2 + 5] - [9]}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{9} + 4h + h^2 - \cancel{9}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{4h + h^2}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{h}(4+h)}{\cancel{h}}$$

$$= \lim_{h \rightarrow 0} 4+h$$

$$= 4+h = 4+(0) = \textcircled{4}$$

(Example 2) Find the IRDC of $f(x) = \sqrt{x}$ at $x=4$.

$$m = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\sqrt{a+h} - \sqrt{a}}{h} \cdot \frac{\sqrt{a+h} + \sqrt{a}}{\sqrt{a+h} + \sqrt{a}}$$

$$= \lim_{h \rightarrow 0} \frac{(a+h) - a}{h(\sqrt{a+h} + \sqrt{a})}$$

$$= \lim_{h \rightarrow 0} \frac{h}{h(\sqrt{a+h} + \sqrt{a})}$$

$$= \lim_{h \rightarrow 0} \frac{1}{\sqrt{a+h} + \sqrt{a}}$$

$$= \frac{1}{\sqrt{a+0} + \sqrt{a}}$$

$$= \frac{1}{2\sqrt{a}} = \frac{1}{2\sqrt{4}} = \frac{1}{4} = \textcircled{\frac{1}{4}}$$

[Example 3] Find the equation of the line of the curve $f(x) = 4 - x^2$ at $x = 1$.

Find slope of tangent line.

$$m = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} = \lim_{h \rightarrow 0} \frac{f(-2a-h)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{[4 - (a+h)^2] - [4 - a^2]}{h} = \lim_{h \rightarrow 0} -2a - h$$

$$= \lim_{h \rightarrow 0} \frac{[4 - (a^2 + 2ah + h^2)] - [4 - a^2]}{h} = -2a - (0)$$
$$= -2a$$
$$= -2(1) = -2$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{4} - a^2 - 2ah - h^2 - \cancel{4} + a^2}{h}$$

$$= \lim_{h \rightarrow 0} \frac{-2ah - h^2}{h}$$

Find equation of tangent line:

$$y = mx + b$$

$$x = 1,$$
$$m = -2$$

need to find y :

$$f(1) = 4 - (1)^2 = 3$$

$$3 = (-2)(1) + b$$

$$3 = -2 + b$$

$$5 = b$$

$$\boxed{y = -2x + 5}$$

[Example 4] A rock breaks loose from the top of a tall cliff. The function modeling the situation is $f(x) = 16x^2$. Find the speed of the falling rock at $x = 1$.

$$m = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{[16(a+h)^2] - [16a^2]}{h}$$

$$= \lim_{h \rightarrow 0} \frac{[16(a^2 + 2ah + h^2)] - [16a^2]}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{16a^2} + 32ah + 16h^2 - \cancel{16a^2}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{32ah + 16h^2}{h}$$

$$= \lim_{h \rightarrow 0} \frac{k(32a + 16h)}{h}$$

$$= \lim_{h \rightarrow 0} 32a + 16h$$

$$= 32a + 16(0)$$

$$= 32a$$

$$= 32(1) = \textcircled{32}$$