

Category	Preparation	Completeness	Organization/Neatness	Timeliness	Total
Points earned					

AP Calculus Problem Set 4

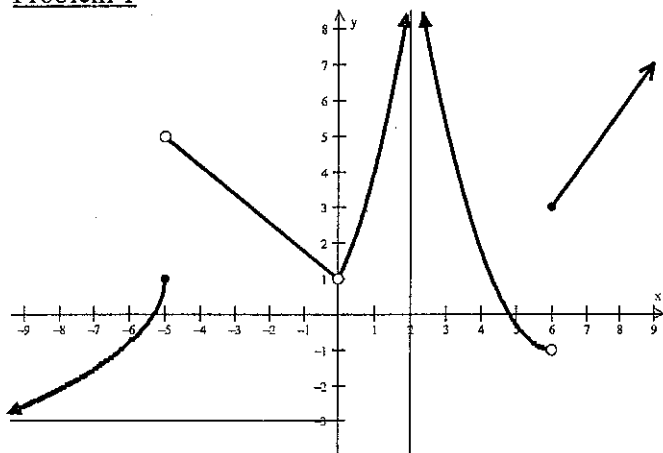
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Upon completion, circle one of the following to assess your current understanding:

Completely understand	Mostly understand	Sort of understand	Don't understand
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Evaluate the given limits and determine the domain and range for each function.

Problem 1



$$\lim_{x \rightarrow -5^-} f(x) = \lim_{x \rightarrow -5^+} f(x) =$$

$$\lim_{x \rightarrow 2^-} f(x) =$$

$$\lim_{x \rightarrow 6^-} f(x) =$$

$$\lim_{x \rightarrow -5^+} f(x) = \lim_{x \rightarrow 0^+} f(x) =$$

$$\lim_{x \rightarrow 2^+} f(x) =$$

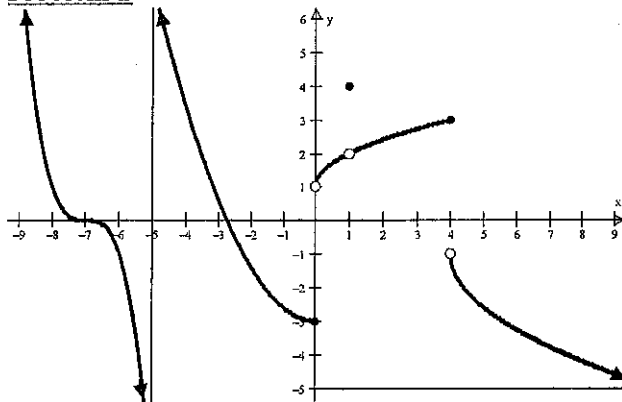
$$\lim_{x \rightarrow 6^+} f(x) =$$

$$\lim_{x \rightarrow -5} f(x) = \lim_{x \rightarrow 0} f(x) =$$

$$\lim_{x \rightarrow 2} f(x) =$$

$$\lim_{x \rightarrow 6} f(x) =$$

Problem 2



$$\lim_{x \rightarrow -5^-} f(x) = \lim_{x \rightarrow -5^+} f(x) =$$

$$\lim_{x \rightarrow 1^-} f(x) =$$

$$\lim_{x \rightarrow 4^-} f(x) =$$

$$\lim_{x \rightarrow -5^+} f(x) = \lim_{x \rightarrow 0^+} f(x) =$$

$$\lim_{x \rightarrow 1^+} f(x) =$$

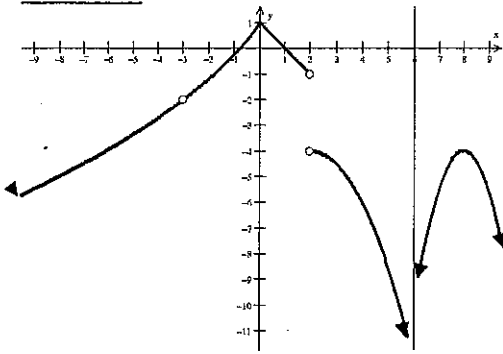
$$\lim_{x \rightarrow 4^+} f(x) =$$

$$\lim_{x \rightarrow -5} f(x) = \lim_{x \rightarrow 0} f(x) =$$

$$\lim_{x \rightarrow 1} f(x) =$$

$$\lim_{x \rightarrow 4} f(x) =$$

Problem 3



$$\lim_{x \rightarrow -3^-} f(x) = \lim_{x \rightarrow -3^+} f(x) =$$

$$\lim_{x \rightarrow 2^-} f(x) =$$

$$\lim_{x \rightarrow 6^-} f(x) = \lim_{x \rightarrow 6^-} f(x) =$$

$$\lim_{x \rightarrow -3^+} f(x) = \lim_{x \rightarrow 0^+} f(x) =$$

$$\lim_{x \rightarrow 2^+} f(x) =$$

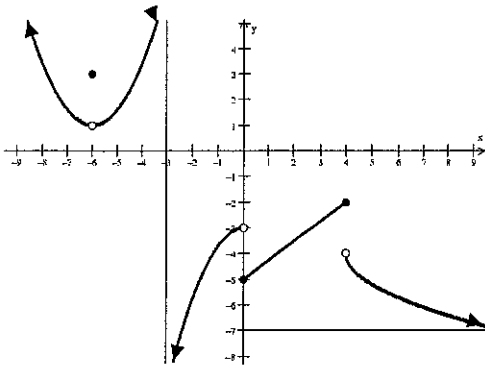
$$\lim_{x \rightarrow 6^+} f(x) = \lim_{x \rightarrow 6^+} f(x) =$$

$$\lim_{x \rightarrow -3} f(x) = \lim_{x \rightarrow 0} f(x) =$$

$$\lim_{x \rightarrow 2} f(x) =$$

$$\lim_{x \rightarrow 6} f(x) =$$

Problem 4



$$\lim_{x \rightarrow -6^-} f(x) = \lim_{x \rightarrow -3^-} f(x) =$$

$$\lim_{x \rightarrow 0^-} f(x) =$$

$$\lim_{x \rightarrow 4^-} f(x) = \lim_{x \rightarrow 4^-} f(x) =$$

$$\lim_{x \rightarrow -6^+} f(x) = \lim_{x \rightarrow -3^+} f(x) =$$

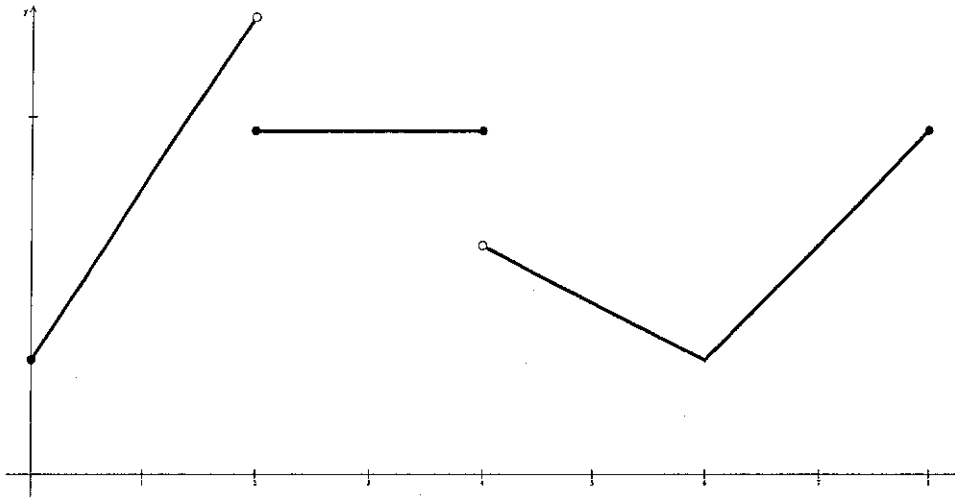
$$\lim_{x \rightarrow 0^+} f(x) =$$

$$\lim_{x \rightarrow 4^+} f(x) = \lim_{x \rightarrow 4^+} f(x) =$$

$$\lim_{x \rightarrow -6} f(x) = \lim_{x \rightarrow -3} f(x) =$$

$$\lim_{x \rightarrow 0} f(x) =$$

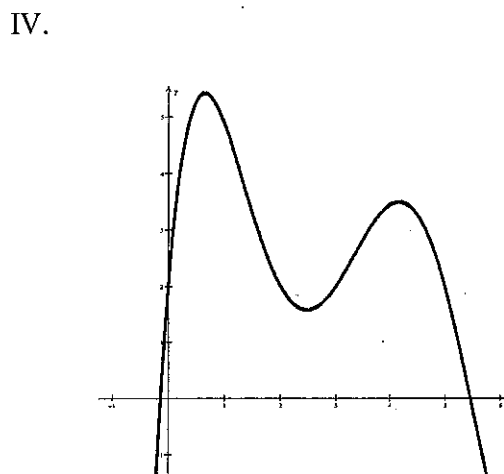
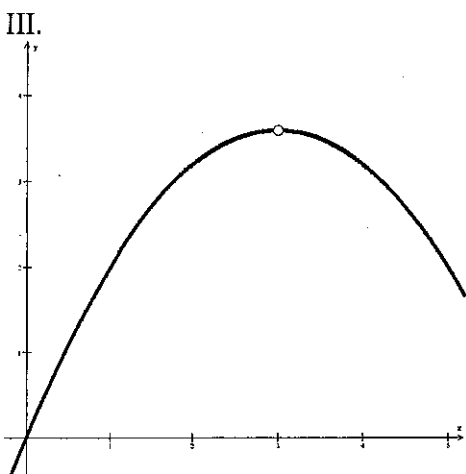
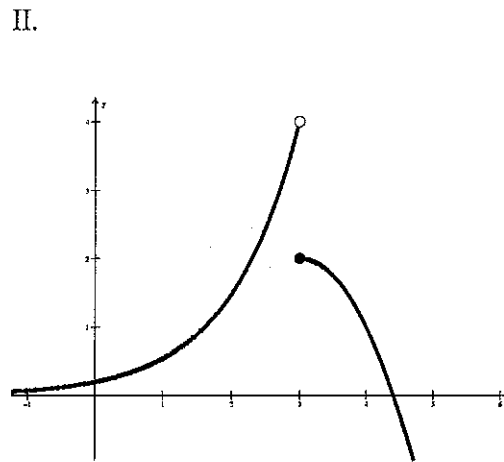
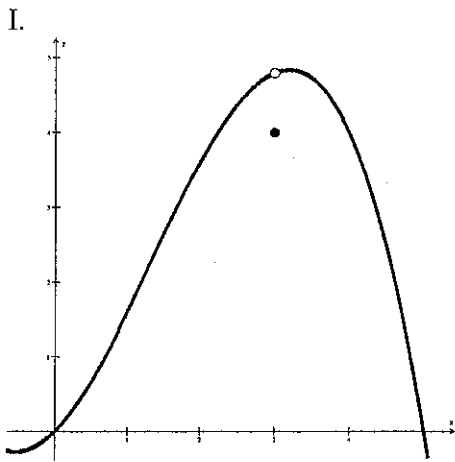
$$\lim_{x \rightarrow 4} f(x) =$$



5. The figure above shows the graph of a function  $f$  with domain  $0 \leq x \leq 8$ . Which of the following statements are true?

- |  |  |  |
|--|--|--|
| I. $\lim_{x \rightarrow 2^-} f(x)$ exists  | IV. $\lim_{x \rightarrow 4^-} f(x)$ exists | VII. $\lim_{x \rightarrow 6^-} f(x)$ exists  |
| II. $\lim_{x \rightarrow 2^+} f(x)$ exists | V. $\lim_{x \rightarrow 4^+} f(x)$ exists  | VIII. $\lim_{x \rightarrow 6^+} f(x)$ exists |
| III. $\lim_{x \rightarrow 2} f(x)$ exists  | VI. $\lim_{x \rightarrow 4} f(x)$ exists   | IX. $\lim_{x \rightarrow 6} f(x)$ exists     |

6. For which of the following does  $\lim_{x \rightarrow 3} g(x)$  exist?



7. Graph a sketch of a function,  $f$ , that has the following characteristics.

$$\lim_{x \rightarrow 1^-} f(x) = 2$$

$$\lim_{x \rightarrow 4} f(x) = 5$$

$$\lim_{x \rightarrow 0} f(x) = \infty$$

$$\lim_{x \rightarrow 1^+} f(x) = 3$$

$$f(4) = 1$$

$$\lim_{x \rightarrow -\infty} f(x) = \infty$$

$\lim_{x \rightarrow 1} f(x)$  does not exist

$$f(1) = 2$$

8. Graph a sketch of a function,  $g$ , that has the following characteristics.

$$\lim_{x \rightarrow 3^-} g(x) = \infty$$

$$\lim_{x \rightarrow 5} g(x) = 2$$

$$\lim_{x \rightarrow \infty} g(x) = 1$$

$$\lim_{x \rightarrow 3^+} g(x) = -\infty$$

$$g(5) = 1$$

$$\lim_{x \rightarrow -\infty} g(x) = -\infty$$

$\lim_{x \rightarrow 3} g(x)$  does not exist

9. Graph a sketch of a function,  $h$ , that has the following characteristics.

$$\lim_{x \rightarrow 2^-} h(x) = -1$$

$$\lim_{x \rightarrow -3^-} h(x) = -\infty$$

$$\lim_{x \rightarrow \infty} h(x) = \infty$$

$$\lim_{x \rightarrow 2^+} h(x) = 0$$

$$\lim_{x \rightarrow -3^+} h(x) = \infty$$

$$\lim_{x \rightarrow -\infty} h(x) = 0$$

$\lim_{x \rightarrow 2} h(x)$  does not exist

$\lim_{x \rightarrow -3} h(x)$  does not exist

$$h(2) = 0$$

For problems 10-14, determine if each statement **must be true**, **could be false**, or **must be false**.

10. If  $\lim_{x \rightarrow \infty} f(x) = 2$ , then the graph of  $f$  has at least one horizontal asymptote.

11. If  $\lim_{x \rightarrow 0^-} f(x)$  exists and  $\lim_{x \rightarrow 0^+} f(x)$  exists, then  $\lim_{x \rightarrow 0} f(x)$  exists.

12. If  $\lim_{x \rightarrow 2} f(x) = 1$ , then  $\lim_{x \rightarrow 2^-} f(x) = 1$  and  $\lim_{x \rightarrow 2^+} f(x) = 1$

13. If the graph of  $f$  has two distinct horizontal asymptotes, then  $\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow -\infty} f(x)$

14. If  $\lim_{x \rightarrow 1^-} f(x) = \infty$ , then  $\lim_{x \rightarrow 1^+} f(x) = -\infty$

## Limit Problems

Find each limit if it exists:

1.  $\lim_{x \rightarrow 3} \frac{5x+11}{x+1}$

2.  $\lim_{x \rightarrow 2} \frac{3x^2 - x - 10}{x^2 - x - 2}$

3.  $\lim_{x \rightarrow 5} \frac{\frac{1}{x} - \frac{1}{5}}{x - 5}$

4.  $\lim_{x \rightarrow 3^+} \frac{1}{x-3}$

5.  $\lim_{x \rightarrow 2} \frac{\sqrt{x} - \sqrt{2}}{x-2}$

6.  $\lim_{x \rightarrow 1} \frac{x^3 + x - 2}{x-1}$

7.  $\lim_{x \rightarrow 2} \frac{x^3 - 8}{x-2}$

8.  $\lim_{x \rightarrow -2} \frac{x^4 - 16}{x+2}$

9.  $\lim_{x \rightarrow 0} \frac{6x^2 + 2x - 17}{4x^2 + 9}$

10.  $\lim_{x \rightarrow 0} \frac{5x+11}{x^2+1}$

Sketch each piecewise function  $f$  and, for the indicated value of  $a$ , find each limit if it exists:

a)  $\lim_{x \rightarrow a^-} f(x)$

b)  $\lim_{x \rightarrow a^+} f(x)$

c)  $\lim_{x \rightarrow a} f(x)$

11.  $f(x) = \begin{cases} 3x & \text{if } x \leq 2 \\ x^2 & \text{if } x > 2 \end{cases} \quad a = 2$

12.  $f(x) = \begin{cases} \frac{x^2 + x}{x} & \text{if } x \neq 0 \\ 2 & \text{if } x = 0 \end{cases} \quad a = 0$

Using a graphing calculator, verify each limit by:

a) creating a table of function values for  $x$  close to  $a$

b) graphing and zooming in on the graph of  $f$  at  $a$

13.  $\lim_{x \rightarrow 0} \frac{x}{\tan x} = 1$

14.  $\lim_{x \rightarrow 3} \frac{x^3 + 2x^2 - 9x - 18}{x-3} = 30$

15.  $\lim_{x \rightarrow 0} \frac{x}{\sin x} = 1$