2.1 Basic Differentiation Rules

Standards: MCD1 MCD1e

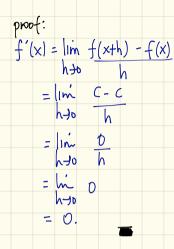
$$\begin{array}{c|c} \hline \textbf{Old} & Derivatives using limit Definition \\ \hline Find f'(x) if f(x) = x^2 + 5 \\ f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{[(x+h)^2 + 5] - [x^2 + 5]}{h} \\ = \lim_{h \to 0} \frac{x^2 + 2xh + h^2 + 5 - x^2 - 5}{h} \\ = \lim_{h \to 0} \frac{x^2 + 2xh + h^2 + 5 - x^2 - 5}{h} \\ = \lim_{h \to 0} \frac{2xh + h^2}{h} = \lim_{h \to 0} \frac{h(2x+h)}{h} \\ = \lim_{h \to 0} \frac{2x + h}{h} \\ = 2x + (0) \\ = 2x \\ \text{Computing this derivative us n't difficult. (Kinda easy but lots of work!)} \end{array}$$

[New] Basic Differentiation Rules What if we considered such functions as $f(x) = x^4 + 3x^2 + 2$ or $g(x) = 5x^5 + 6x^3 + x^4 + 3x^2$?

Computing the derivative of these functions is EXTREMELY DIFFICULT & TEDIONS using the "limit method". Fortunately, several formulas have been developed to simplify the process of differentiation.

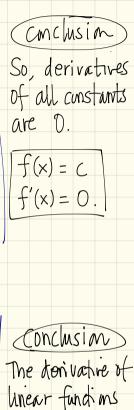
A Constants

Let's consider the function: f(x) = c, where C is some real number. Find f'(x).



B Linear Functions Let's consider the function: f(x)=nx, where n e IR. Find f'(x).

proof: $f'(x) = \lim_{h \to 0} \frac{f(xth) - f(x)}{h}$ $= \lim_{h \to 0} \frac{[n(xth)] - [nx]}{h}$ $= \lim_{h \to 0} \frac{hxt - hx}{h}$ $= \lim_{h \to 0} \frac{hxt}{h}$



is the coefficient of the variable.

f(x) = nxf'(x) = n

BINOMIAL THEOREM $(x+h)^n = \sum_{k=1}^n \binom{n}{k} q^{n-k} b^k$ $= \chi^{n} + {\binom{n}{1}} \chi^{n-1} h + {\binom{n}{2}} \chi^{n-2} h^{2} + {\binom{n}{3}} \chi^{n-3} h^{3} + \cdots + {\binom{n}{n-1}} \chi^{n-1} h^{n} h^{n}$ $= X^{n} + h X^{n-1} h + \frac{n(n-1)}{2!} x^{n-2} h^{2} + n (n-1)(n-2) X^{n-3} h^{3} + \cdots$ +nxh⁻¹th Where, $\binom{n}{k} = \frac{n}{k!(n-k)!}$

C Power Function Where NER. Let's consider the function: $f(x) = x^n$, proof: $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \to 0} \frac{[(x+h)^{n}] - [x^{n}]}{h}$ = $\lim_{h \to 0} \frac{[(x+h)^{n}] - [x^{n}]}{h}$ = $\lim_{h \to 0} \frac{[x^{n} + nx^{n-1}h + \frac{n(n-1)}{2!}x^{n-2}h^{2} + \frac{n(n-1)(n-2)}{3!}x^{n-3}h^{3} + \dots + h^{n}f[x^{n}]}{3!}$ $=\lim_{h \to 0} h x^{n-1} h + \frac{h(n-1)}{2!} x^{n-2} h + \frac{h(n-1)(n-2)}{3!} x^{n-3} h^{2} + \dots + h^{n-1}$ h-)0 = | in n x n - 1 $= n x^{n-1}$ Conclusin $f(x) = x^n$ $f'(x) = nx^{n-1}$

Examples Find derivatives

 $\begin{array}{c} (4) f(x) = \frac{1}{x^{5}} = x^{-5} \\ f'(x) = -5x^{-6} \\ f'(x) = -\frac{1}{3}x^{-\frac{1}{5}} \\ \end{array}$