2.2 Calculating Tangent Lines

Standards:

MCD1

MCD1a

MCD1b

MCD1c

$\boxed{\text{old}}$ → we "guessed" or estimated values of slopes of tangent lines on the asus of numerical evidence

$\boxed{\text{new}}$ → we are going to algebraically evaluate the values of the tangent lines

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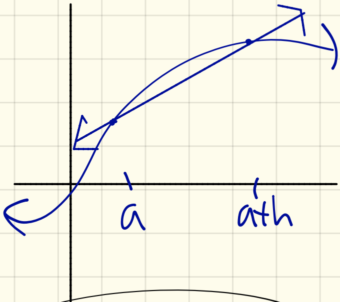
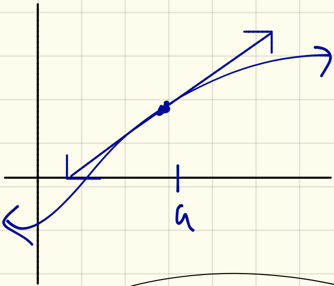
Let's remember the following formulas:



Slope of secant line

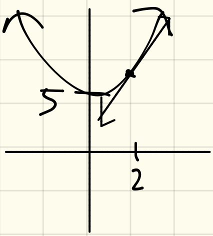
$$m = \frac{f(a+h) - f(a)}{h}$$



Slope of tangent line

$$m = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

[Example 1a] Estimate the slope of the tangent line for $f(x) = x^2 + 5$ when $x = 2$



x	$\frac{f(2) - f(x)}{2 - x}$
1.9	3.9
1.99	3.99
1.999	3.999
2.001	4.001
2.01	4.01
2.1	4.1

2   4

[Example 1b] Evaluate the slope of the tangent line for $f(x) = x^2+5$ when $x=2$.

$$m = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$$

$$= \lim_{h\to 0} \frac{f(2+h)-f(2)}{h}$$

$$= \lim_{h\to 0} \frac{[(2+h)^2+5]-[9]}{h}$$

$$= \lim_{h\to 0} \frac{(4+4h+h^2+5)-(9)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{9}+4h+h^2-\cancel{9}}{h}$$

$$= \lim_{h\to 0} \frac{4h+h^2}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(4+h)}{\cancel{h}}$$

$$= \lim_{h\to 0} 4+h$$

$$- 4+h = 4+(0) = 4$$

[Example 2] Find the IROC of $f(x)=\sqrt{x}$ at $x=4$.

$$m = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{(a+h)}-\sqrt{a}}{h} \cdot \frac{\sqrt{a+h}+\sqrt{a}}{\sqrt{a+h}+\sqrt{a}}$$

$$= \lim_{h\to 0} \frac{(a+h)-a}{h(\sqrt{a+h}+a)}$$

$$= \lim_{h\to 0} \frac{h}{h(\sqrt{a+h}+\sqrt{a})}$$

$$= \lim_{h\to 0} \frac{1}{\sqrt{a+h}+\sqrt{a}}$$

$$= \frac{1}{\sqrt{a+0}+\sqrt{a}}$$

$$= \frac{1}{2\sqrt{a}} = \frac{1}{2\sqrt{4}} = \frac{1}{2(2)} = \frac{1}{4}$$

[Example 3] Find the equation of the line of the curve $f(x) = 4-x^2$ at $x=1$.

Find slope of tangent line.

$$m = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$$

$$= \lim_{h\to 0} \frac{[4-(a+h)^2]-[4-a^2]}{h}$$

$$= \lim_{h\to 0} \frac{[4-(a^2+2ah+h^2)]-[4-a^2]}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{4}\cancel{-a^2}-2ah-h^2\cancel{-4}\cancel{+a^2}}{h}$$

$$= \lim_{h\to 0} \frac{-2ah-h^2}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(-2a-h)}{\cancel{h}}$$

$$= \lim_{h\to 0} -2a-h$$

$$= -2a-(0)$$

$$= -2a$$

$$= -2(1) = -2$$

Find equation of tangent line:

$y = mx+b$

$x = 1,$

$m = -2$

need to find y:

$f(1) = 4-(1)^2 = 3$

$3 = (-2)(1)+b$

$3 = -2+b$

$5 = b$

$$\boxed{y = -2x+5}$$

This was created by Keenan Xavier Lee, 2013. See my website for more information, lee-apcalculus.weebly.com.

[Example 4] A rock breaks loose from the top of a tall cliff. The function modeling the situation is $f(x) = 16x^2$. Find the speed of the falling rock at $x = 1$.

$$m = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$$

$$= \lim_{h\to 0} \frac{[16(a+h)^2] - [16a^2]}{h}$$

$$= \lim_{h\to 0} \frac{[16(a^2+2ah+h^2)] - [16a^2]}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{16a^2} + 32ah + 16h^2 - \cancel{16a^2}}{h}$$

$$= \lim_{h\to 0} \frac{32ah + 16h^2}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(32a + 16h)}{\cancel{h}}$$

$$= \lim_{h\to 0} 32a + 16h$$

$$= 32a + 16(0)$$

$$= 32a$$

$$= 32(1) = 32$$