

2.2 Product Rule

Standard:

MCD1e



[Old] Basic Differentiation Rules

$$\textcircled{1} f(x) = x^4 + 5 \\ f'(x) = 4x^3$$

$$\textcircled{2} f(x) = \frac{1}{x^2} = x^{-2} \\ f'(x) = -2x^{-3}$$

$$\textcircled{3} f(x) = \sqrt{10} + \sqrt{x} + \frac{5}{x^3} \\ = \sqrt{10} + x^{\frac{1}{2}} + 5x^{-3} \\ f'(x) = \frac{1}{2}x^{-\frac{1}{2}} - 15x^{-4}$$

[New] Product Rule

Recall: What is the derivative of $f(x) = 2x^2 + 5x$

$$f'(x) = 4x + 5$$

$$\bullet \frac{d}{dx}(x^4 + 3x^2 + e^2) = 4x^3 + 6x$$

Let's consider the function: $f(x) = (x+2)(5x)$.

What happens when we have a product of 2 functions?

One might be tempted to just take the derivative of each & then multiply just like one would for sum & difference of 2 functions.

However, this is WRONG!

Let's prove what we need to do in this case.

PRODUCT RULE Let's consider the function: $k(x) = f(x) \cdot g(x)$.

Find the derivative.

proof:

$$k'(x) = \lim_{h \rightarrow 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{f(x+h)g(x+h) - f(x+h)g(x) + f(x+h)g(x) - f(x)g(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{f(x+h)[g(x+h) - g(x)] + g(x)[f(x+h) - f(x)]}{h}$$

$$= \lim_{h \rightarrow 0} \frac{f(x+h)[g(x+h) - g(x)]}{h} + \lim_{h \rightarrow 0} \frac{g(x)[f(x+h) - f(x)]}{h}$$

$$= \lim_{h \rightarrow 0} f(x+h) \cdot \lim_{h \rightarrow 0} \frac{g(x+h) - g(x)}{h} + \lim_{h \rightarrow 0} g(x) \cdot \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$= f(x) \cdot g'(x) + g(x) \cdot f'(x)$$

$$\boxed{\text{Product Rule}} \quad f(x)g(x) = f(x) \cdot g'(x) + g(x) \cdot f'(x)$$

$$\text{[Example 1]} \quad \frac{d}{dx}(x+2)(5x) ?$$

$$\begin{aligned} f \cdot g' + g \cdot f' &= (x+2)(5) + (5x)(1) \\ &= 5x+10 + 5x \\ &= 10x+10 \end{aligned}$$

$$\text{[Example 2]} \quad f(x) = (x^2+1)(x^3+1)$$

$$\begin{aligned} f'(x) &= (x^2+1)(3x^2) + (x^3+1)(2x) \\ &= 3x^4 + 3x^3 + 2x^4 + 2x \\ &= 5x^4 + 3x^3 + 2x \end{aligned}$$