2.4 Differentiability

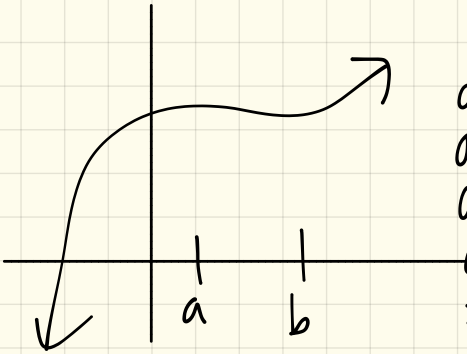
Standards:

MCD1

MCD1d

Old Derivatives

The process of finding derivatives of functions is called differentiation.



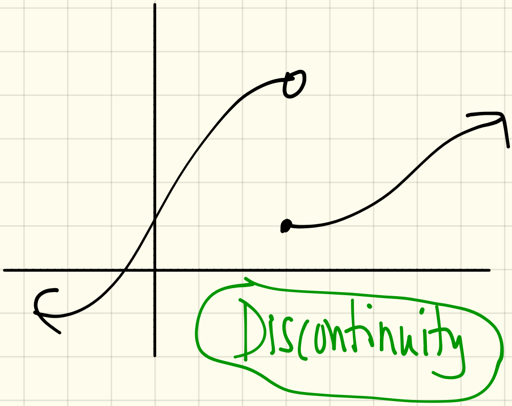
A function is differentiable at x if the derivative exists at x and differentiable on an open interval (a,b) if its differentiable at every point in the interval.

New Differentiability

How can a function fail to be differentiable?

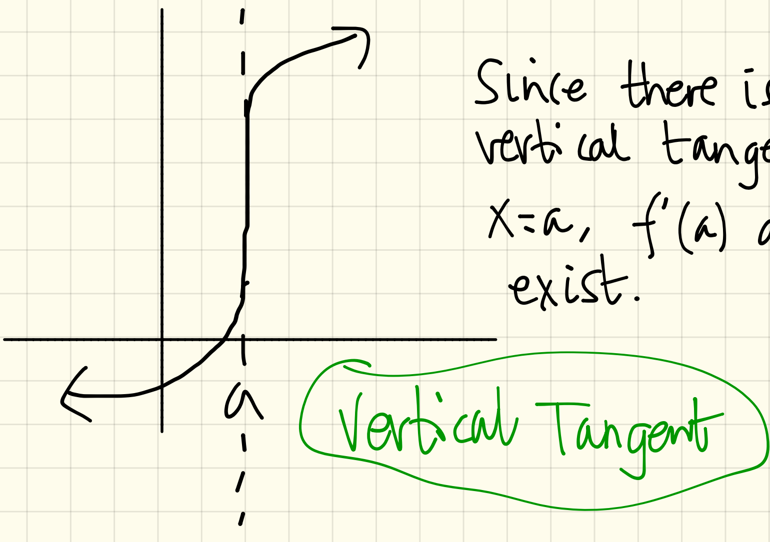
1) Discontinuity
2) Vertical Tangent
3) Sharp Corner.

$\boxed{1}$ If $f(x)$ is not continuous at $x=a$, then $f(a)$ does <u>not</u> exist.



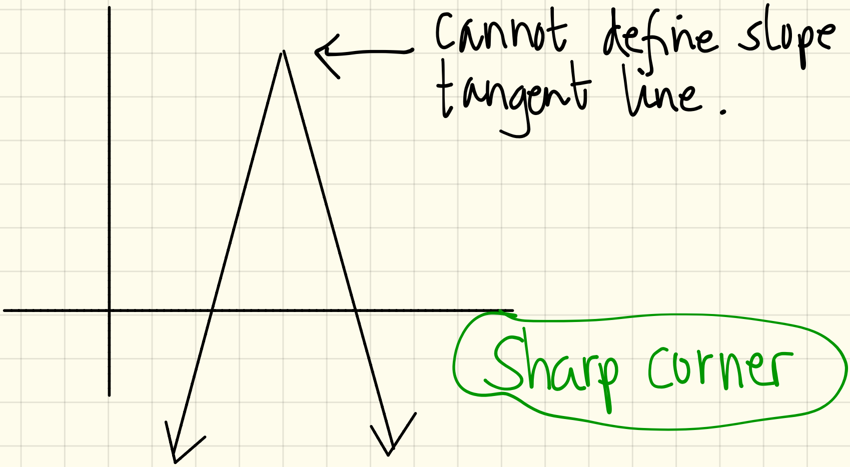
$\lim\limits_{x \to a}$ does not exist because if f is not continuous at $x=a$, then we can't make sense of the "slope of the tangent line at $x=a$".

$\boxed{2}$ If the slope becomes "infinitely steep", then the derivative does not exist.



Since there is a vertical tangent at $x=a$, $f'(a)$ does not exist.

$\boxed{3}$ If $f(x)$ has a "sharp corner" at $x=a$, then $f'(a)$ does not exist.

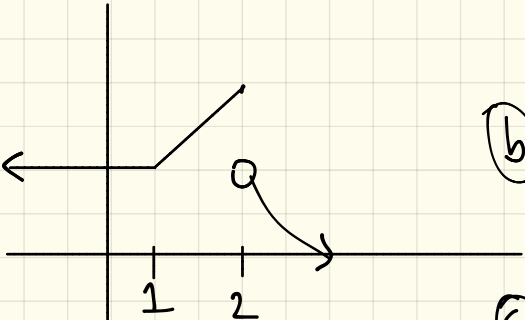


Theorems

$\boxed{A}$ If f is differentiable at (a), then f is continuous at (a).

$\boxed{B}$ If f is not continuous at (a), then f is not differentiable at (a).

[Example 1]



ⓐ What x-values are not differentiable?

$x=1, \ x=2$

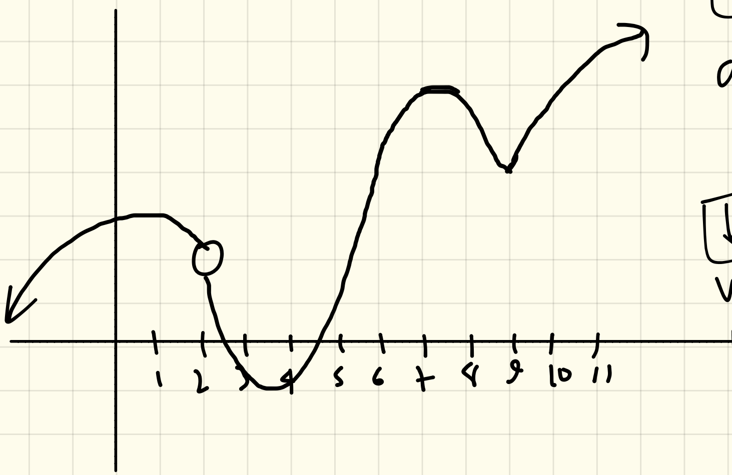
ⓑ What x-values is f not continuous?

$x=2$

ⓒ Where on f does the limit not exist?

$x=2$

[Example 2]



$\boxed{a}$ Determine which x-values are not continuous.

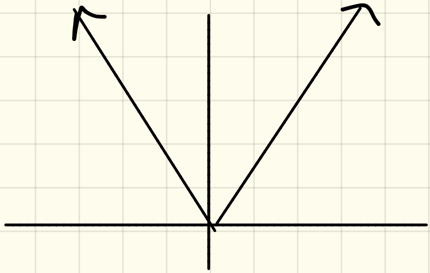
$x=2$

$\boxed{b}$ Determine which x-values not differentiable

$x=2, x=9$

Let's consider $f(x) = |x|$. Is $x=0$ differentiable?

$$f(x) = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$



$$m = \lim_{h \to 0^-} \frac{f(a+h) - f(a)}{h}$$

$$= \lim_{h \to 0^-} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h \to 0^-} \frac{|0+h| - |0|}{h}$$

$$= \lim_{h \to 0^-} \frac{-(0+h) - (-0)}{h}$$

$$= \lim_{h \to 0^-} \frac{\cancel{-0} - h + \cancel{0}}{h}$$

$$= \lim_{h \to 0^-} \frac{-h}{h}$$

$$= \lim_{h \to 0^-} -1$$

$$= -1$$

$$m = \lim_{h \to 0^+} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h \to 0^+} \frac{|0+h| - |0|}{h}$$

$$= \lim_{h \to 0^+} \frac{0+h-0}{h}$$

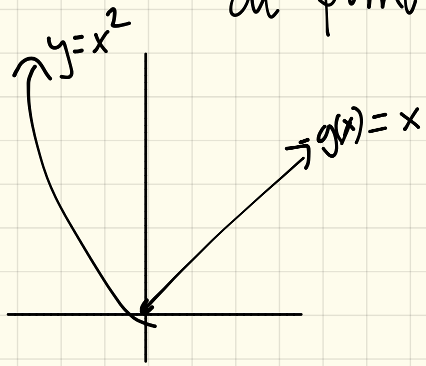
$$= \lim_{h \to 0} \frac{h}{h}$$

$$= \lim_{h \to 0} 1 = 1$$

Conclusion The rapid change in slope from -1 to 1 at $x=0$ makes this function not differentiable.

The change needs to be gradual, not rapid.

[Example 3] Consider the graph & show that the f(x) is not differentiable at point P.



$$f'(x) = \lim_{h \to 0^-} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0^-} \frac{[(x+h)^2] - [x^2]}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2}{h}$$

$$= \lim_{h \to 0} \frac{2xh + h^2}{h}$$

$$= \lim_{h \to 0} \frac{h(2x+h)}{h}$$

$$= \lim_{h \to 0} 2x + h = 2x$$

$$g'(x) = \lim_{h \to 0^+} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0^+} \frac{(x+h) - (x)}{h}$$

$$= \lim_{h \to 0} \frac{h}{h}$$

$$= \lim_{h \to 0} 1 = 1$$

$$g'(0) = 1$$

$$f'(0) = 0$$

Since there is a rapid change from 0 to 1 at $x=0$, then the point $(0,0)$ is not differentable.