2.5 Derivatives of Trigonometric Functions

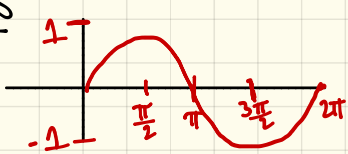
Standard:

MCD1e

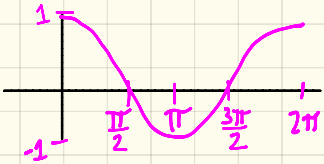
Old Trig Review

Trig Functions

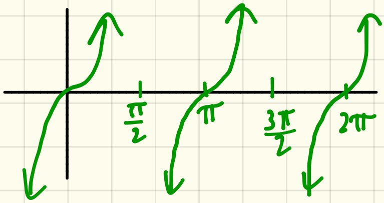
- $\sin(x)$



- $\cos(x)$



- $\tan(x)$



Reciprocal Functions

- $\csc(x) = \frac{1}{\sin x}$
- $\sec(x) = \frac{1}{\cos x}$
- $\cot(x) = \frac{1}{\tan(x)}$

Trig Value Chart – Memorize

Degrees	Radians	Sin(x)	Cos(x)
0	0, π	0	1
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
45°	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
90°	$\frac{\pi}{2}$	1	0

[Examples] Evaluate Trig Functions.

1. $\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$

2. $\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

3. $\sin\left(\frac{5\pi}{4}\right) =$

$= \sin(225)$

$= -\sin(45)$

$= -\frac{\sqrt{2}}{2}.$

$\boxed{\text{new}}$ Trig Derivatives

Let's consider $f(x) = \sin(x)$. Find the derivative.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\sin(x+h) - \sin(x)}{h}$$

Sidenote: Double Angle Formula

$\sin(x+h) = \sin x \cosh + \cos x \sin h$

$$= \lim_{h\to 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$$

$$= \lim_{h\to 0} \frac{\sin x \cos h - \sin x + \cos x \sin h}{h}$$

$$= \lim_{h\to 0} \frac{\sin x(\cos h - 1) + \cos x \sin h}{h}$$

$$= \lim_{h\to 0} \frac{\sin x(\cos h - 1)}{h} + \lim_{h\to 0} \frac{\cos x \sin h}{h}$$

$$= \lim_{h\to 0} \sin x \cdot \lim_{h\to 0} \frac{\cos h - 1}{1} + \lim_{h\to 0} \cos x \cdot \lim_{h\to 0} \frac{\sin h}{h}$$

$$= \sin(x) \cdot \frac{\cos(0) - 1}{1} + \cos x \cdot 1 \text{ (graphically)}$$

$$= (\sin x \cdot 0) + (\cos x \cdot 1)$$

$$= \cos x$$

■

Conclusion: $\frac{d}{dx}(\sin x) = \cos x$.

Trig Derivatives — Memorize

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\cot x) = -\csc^2 x$$

[Examples] Find the derivatives of each function.

$PR = f \cdot g' + g \cdot f'$

① $f(x) = x^3 \sin x$

$f'(x) = (x^3)(\cos x) + (\sin x)(3x^2)$

$= x^3 \cos x + 3x^2 \sin x$

$= x^2(x\cos x + 3\sin x).$

$QR = \dfrac{g \cdot f' - f \cdot g'}{g^2}$

② $f(x) = \dfrac{x^3}{\cos x}$

$f'(x) = \dfrac{(\cos x)(3x^2) - (x^3)(-\sin x)}{(\cos x)^2}$

$= \dfrac{3x^2 \cos x + x^3 \sin x}{\cos^2 x}$

$= \dfrac{x^2(3\cos x + x\sin x)}{\cos^2 x}.$

③ $f(x) = x\tan x$

$f'(x) = (x)(\sec^2 x) + (\tan x)(1)$

$= x\sec^2 x + \tan x.$

⑤ $f(x) = \cos x \sin x$

$f'(x) = (\cos x)(\cos x) + (\sin x)(-\sin x)$

$= \cos^2 x - \sin^2 x$

⑥ $f(x) = x^3 + \csc x$

$f'(x) = 3x^2 - \csc x \cot x$

⑦ $f(x) = \sqrt{2} + \sec x$

$f'(x) = \sec x \tan x$

⑧ $f(x) = 6\cot x$

$f'(x) = -6\csc^2 x.$

9) $f(x) = \dfrac{x^2 \sin x}{\cos x}$

$$f'(x) = \frac{(\cos x)\left[(x^2)(\cos x) + (\sin x)(2x)\right] - (x^2 \sin x)(-\sin x)}{(\cos x)^2}$$

$$= \frac{\cos x\left[x^2 \cos x + 2x \sin x\right] + x^2 \sin^2 x}{\cos^2 x}$$

$$= \frac{x^2 \cos^2 x + 2x \cos x \sin x + x^2 \sin^2 x}{\cos^2 x}$$

$$= \frac{x^2 \cancel{\cos^2} x}{\cancel{\cos^2} x} + \frac{2x \cancel{\cos} x \sin x}{\cos^{\cancel{2}\ 1} x} + \frac{x^2 \sin^2 x}{\cos^2 x}$$

$$= x^2 + \frac{2 \sin x}{\cos x} + \frac{x^2 \sin^2 x}{\cos^2 x}.$$