2.7 Implicit Differentiation

Standards:

MCD2

MCD2b

$\boxed{\text{Old}}$ chain rule

① $f(x) = (x^2+3)^4$

$f'(x) = 4(x^2+3)^3(2x)$

$= 8x(x^2+3)^3$

② $f(x) = \sin(4x)$

$f'(x) = \cos(4x)\cdot(4)$

$= 4\cos(4x)$

need PR: $f\cdot g' + g\cdot f'$

③ $f(x) = x(x^3+5x)^3$

$f'(x) = (x)\left[3(x^3+5x)^2\cdot(3x^2+5)\right] + (x^3+5x)^3\cdot(1)$

$= x\left[(9x^2+15)(x^3+5x)^2\right] + (x^3+5x)^3$

$\boxed{\text{new}}$ Implicit Differentiation

Let's consider $f(x) = (1+x^5)^9$.

Then, $f'(x) = 9(1+x^5)^8\cdot(5x^4)$

Now, let's consider that we <u>didn't</u> have a formula for the inside function. Let's say knew that y was a function of x.

So, $\frac{d}{dx}(y^9)$.

Basically, this is what happened...

Let $y = (1+x^5)$

$$\frac{d}{dx}(1+x^5)^9 = \frac{d}{dx}y^9$$

So we are going to "act as if" we know that the variable of the function is x.

Now let's take the derivative of y^9.

$$\frac{d}{dx}y^9 = 9y^8 \cdot y'.$$

Isn't that the same as...

Remember $y = (1+x^5)^9$

$$\frac{d}{dx}(1+x^5)^9 = 9(1+x^5)^8 \cdot (5x^4)$$

Implicit Differentiation can be used to find y' in equations involving x's & y's, without solving for y.

[Examples] Find the derivatives.

① $\frac{d}{dx}(x^2+y^2=25)$

$2x+2y\cdot y'=0$

$2y\cdot y'=-2x$

$y'=\frac{-2x}{2y}$

$\boxed{y'=-\frac{x}{y}}$

② $\frac{d}{dx}(4x^2+9y^2=36)$

$8x+18y\cdot y'=0$

$18y\cdot y'=-8x$

$y'=\frac{-8x}{18y}$

$\boxed{y'=-\frac{4x}{9y}}$

③ $\frac{d}{dx}\left(\frac{1}{x}+\frac{1}{y}=1\right)$

Rewrite...

$x^{-1}+y^{-1}=1$

$-1x^{-2}-1y^{-2}\cdot y'=0$

$-x^{-2}-y^{-2}\cdot y'=0$

$-y^{-2}\cdot y'=x^{-2}$

$y'=\frac{x^{-2}}{-y^{-2}}$

$\boxed{y'=\frac{-y^2}{x^2}}$

④ $\frac{d}{dx}(xy+2x+3x^2=4)$

$[(x)(1)\cdot y'+(y)(1)]+2+6x=0$

$xy'+y+2+6x=0$

$xy'=-y-2-6x$

$\boxed{y'=\frac{-y-2-6x}{x}}$

⑤ $\frac{d}{dx}\left(x^2y + xy^2 = 3x\right)$

$$\left[(x^2)\cdot(1)y' + (y)(2x)\right] + \left[(x)\cdot(2y)y' + (y^2)(1)\right] = 3$$

$$x^2y' + 2xy + 2xyy' + y^2 = 3$$

$$x^2y' + 2xyy' + 2xy + y^2 = 3$$

$$x^2y' + 2xyy' = 3 - 2xy - y^2$$

$$y'(x^2 + 2xy) = 3 - 2xy - y^2$$

$$\boxed{y' = \frac{3 - 2xy - y^2}{x^2 + 2xy}}$$

[Example 6] Find the slope of the curve at the indicated point.

$x^2 + y^2 = 13$ at $(-2,3)$

$$2x + 2y \cdot y' = 0$$

$$2yy' = -2x$$

$$y' = \frac{-2x}{2y}$$

$$y' = \frac{-x}{y}$$

$$y'(-2,3) = \frac{-(-2)}{3} = \frac{2}{3}$$ ← slope of tangent line.

Homework page 162: 1-8, 9-10, 17-18.