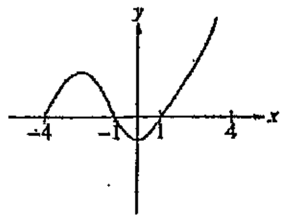
AP RESOURCE QUESTIONS
SUMMER INSTITUTE

1. MC '93 AB 9

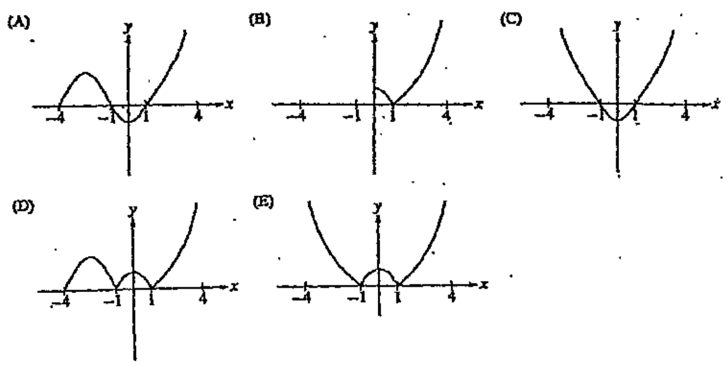
   If h is the function given by $h(x) = f(g(x))$, where $f(x) = 3x^2 - 1$ and $g(x) = |x|$, then $h(x)$

   (A) $3x^3 - |x|$ (B) $|3x^2 - 1|$ (C) $3x^2|x| - 1$ (D) $3|x| - 1$ (E) $3x^2 - 1$

2. MC ' 93 AB 40



   The graph of *y=(x)* is shown in the figure above. Which of the following could be the graph of $y = f(|x|)$?



3. MC '03 BC 76

   The graph of the function *f* is shown above. Which of the following statements must be false?

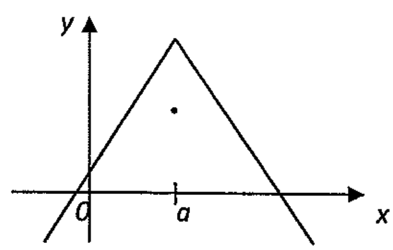
   (A) $f(a)$ exists.

   (B) $f(x)$ is defined for $0 < x < a$.

   (C) f is not continuous at $x = a$.

   (D) $\lim_{x \to a} f(x)$ exists.

   (E) $\lim_{x \to a} f'(x)$ exists.



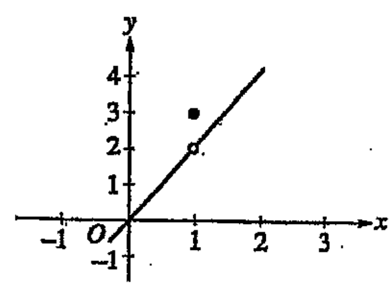
Graph of f

4. MC '03 BC 81

   The graph of the function *f* is shown in the figure above. The value of $\lim_{x \to 1} \sin(f(x))$ is

   (A) 0.909 (B) 0.841 (C) 0.141

   (D) -0.416 (E) nonexistent



Graph of f

5. '70 AB2

A function f is defined on the closed interval from -3 to 3 and has the graph shown below.

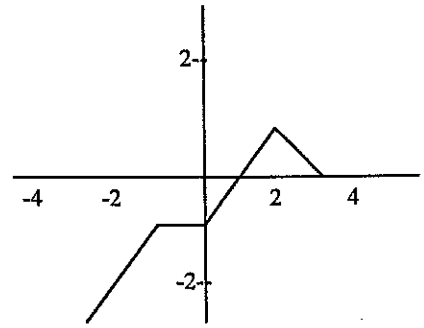
(a) On the axes provided sketch the entire graph of $y=|f(x)|$.

(b) On the axes provided sketch the entire graph of $y=f(|x|)$.

(c) On the axes provided sketch the entire graph of $y=f(-x)$.

(d) On the axes provided sketch the entire graph of $y=f\left(\frac{1}{2}x\right)$.

(e) On the axes provided sketch the entire graph of $y=f(x-1)$.



6. '82 AB2

Given that f is a function defined by $f(x)=\dfrac{x^3-x}{x^3-4x}$.

(a) Find $\lim_{x\to 0} f(x)$.

(b) Find the zeroes of f.

(c) Write an equation for each vertical and each horizontal asymptote to the graph of f.

(d) Describe the symmetry of the graph of f.

(e) Using the information found in parts (a), (b), (c), and (d), sketch the graph of f on the axes provided.

7. MC '93 AB 35

If the graph of $y=\dfrac{ax+b}{x+c}$ has a horizontal asymptote $y=2$ and a vertical asymptote $x=-3$, then $a+c=$

(A) -5 (B) -1 (C) 0 (D) 1 (E) 5

8. MC '93 AB 5

If the function f is continuous for all real numbers and if $f(x)=\dfrac{x^2-4}{x+2}$ when $x\neq -2$, then $f(-2)=$

(A) -4 (B) -2 (C) -1 (D) 0 (E) 2

9. MC '73 AB 23

$\lim_{h\to 0}\frac{1}{h}\ln\left(\frac{2+h}{2}\right)$ is

(A) e^2 (B) 1 (C) $\frac{1}{2}$ (D) 0 (E) nonexistent

10. MC '72 AB 17

Let f be defined as follows, where $a \neq 0$.

$f(x) = \frac{x^2-a^2}{x-a}$, for $x \neq a$,

0, for $x = a$.

Which of the following are true about f?

I. $\lim_{x \to a} f(x)$ exists.

II. $f(a)$ exists.

III. $f(x)$ is continuous at $x = a$.

(A) None (B) I only (C) II only (D) I and II only (E) I, II and III

11. MC '03 BC 79

The table gives values of f, f', g, and g' at selected values of x. If $h(x) = f(g(x))$, then $h'(1) =$

(A) 5 (B) 6 (C) 9 (D) 10 (E) 12

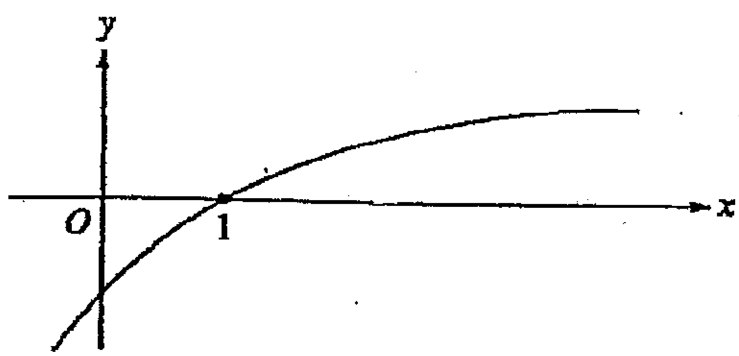
x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
-1	6	5	3	-2
1	3	-3	-1	2
3	1	-2	2	3

12. MC '93 BC 18

If $e^{f(x)} = 1 + x^2$, then $f'(x) =$

(A) $\frac{1}{1+x^2}$ (B) $\frac{2x}{1+x^2}$ (C) $2x(1+x^2)$ (D) $2x\left(e^{1+x^2}\right)$ (E) $2x\ln(1+x^2)$

13. MC '98 AB 17



The graph of a twice-differentiable function f is shown in the figure above. Which of the following is true?

(A) $f(1) < f'(1) < f''(1)$

(B) $f(1) < f''(1) < f'(1)$

(C) $f'(1) < f(1) < f''(1)$

(D) $f''(1) < f(1) < f'(1)$

(E) $f''(1) < f'(1) < f(1)$

14. 1979 AB-6

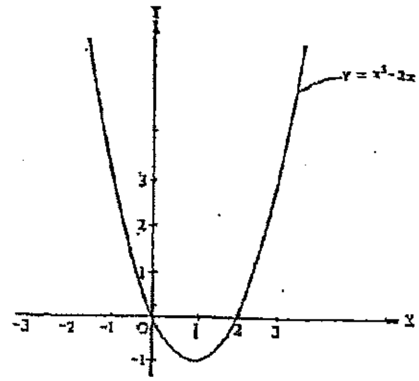
The curve in the figure represents the graph of f, where $f(x) = x^2 - 2x$ for all real numbers x.

(a) On the axes provided, sketch the graph of $y = |f(x)|$.

(b) Determine whether the derivative of $|f(x)|$ exists at $x = 0$. Justify your answer.

(c) On the axes provided, sketch the graph of $y = f(|x|)$.

(d) Determine whether $f(|x|)$ is continuous at $x = 0$. Justify your answer.



15. 1977 AB-4, BC-2

Let f and g and their inverses f^{-1} and g^{-1} be differentiable functions and let the values of f, g, and the derivatives f' and g' at $x = 1$ and $x = 2$ be given by the table below.

Determine the value of each of the following.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	3	2	5	4
2	2	π	6	7

(a) The derivative of $f + g$ at $x = 2$

(b) The derivative of fg at $x = 2$

(c) The derivative of $\frac{f}{g}$ at $x = 2$

(d) $h'(1)$ where $h(x) = f(g(x))$

(e) The derivative of g^{-1} at $x = 2$

16. 1999: AB-6

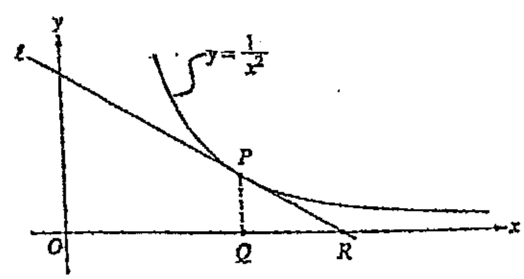
In the figure above, line ℓ is tangent to the graph of $y = \frac{1}{x^2}$ at point P, with coordinates $\left(w, \frac{1}{w^2}\right)$, where $w > 0$. Point Q has coordinates $(w, 0)$. Line ℓ crosses the x-axis at point R, with coordinates $(k, 0)$.

(a) Find the value of k when $w = 3$.

(b) For all $w > 0$, find k in terms of w.

(c) Suppose w is increasing at the constant rate of 7 units per second. When $w = 5$, what is the rate of change of k with respect to time?

(d) Suppose that w is increasing at the constant rate of 7 units per second. When $w = 5$, what is the rate of change of the area of ΔPQR with respect to time? Determine whether the area is increasing or decreasing at this instant.



17. MC '93 AB 18

If $f(x) = \sin\left(\frac{x}{2}\right)$, then there exists a number c in the interval $\frac{\pi}{2} < x < \frac{3\pi}{2}$ that satisfies the conclusion of the Mean Value Theorem. Which of the following could be c?

(A) $\frac{2\pi}{3}$ (B) $\frac{3\pi}{4}$ (C) $\frac{5\pi}{6}$ (D) π (E) $\frac{3\pi}{2}$

18. MC '93 AB 16

The slope of the line normal to the graph of $y = 2\ln(\sec x)$ at $x = \frac{\pi}{4}$ is

(A) -2 (B) $-\frac{1}{2}$ (C) $\frac{1}{2}$ (D) 2 (E) nonexistent

19. 2004 AB/BC-4

Consider the curve given by $x^2 + 4y^2 = 7 + 3xy$.

(a) Show that $\frac{dy}{dx} = \frac{3y-2x}{8y-3x}$.

(b) Show that there is a point *P* with *x*-coordinate 3 at which the line tangent to the curve at P is horizontal. Find the *y*-coordinate of *P*.

(c) Find the value of $\frac{d^2y}{dx^2}$ at the point *P* found in part (b). Does the curve have a local maximum, a local minimum, or neither at the point *P*? Justify your answer.

20. MC '69 AB 45

If $\frac{d}{dx}(f(x)) = g(x)$ and $\frac{d}{dx}(g(x)) = f(x^2)$, then $\frac{d^2}{dx^2}(f(x^3)) =$

(A) $f(x^6)$
(B) $g(x^3)$
(C) $3x^2g(x^3)$
(D) $9x^4f(x^6) + 6xg(x^3)$
(E) $f(x^6) + g(x^3)$

21. 1995 AB-3

Consider the curve defined by $-8x^2 + 5xy + y^3 = -149$.

(a) Find $\frac{dy}{dx}$.

(b) Write an equation for the line tangent to the curve at the point (4, -1).

(c) There is a number *k* so that the point (4.2, *k*) is on the curve. Using the tangent line found in part (b), approximate the value of *k*.

(d) Write an equation that can be solved to find the actual value of *k* so that the point (4.2, *k*) is on the curve.

(e) Solve the equation found in part (d) for the value of *k*.

22. 1981 AB-3, BC-1

Let *f* be the function defined by $f(x) = 12x^{\frac{2}{3}} - 4x$.

(a) Find the intervals on which *f* is increasing.

(b) Find the *x*- and *y*-coordinates of all relative maximum points.

(c) Find the *x*- and *y*-coordinates of all relative minimum points.

(d) Find the intervals on which *f* is concave downward.

(e) Using the information found in parts (a), (b), (c) and (d), sketch the graph of *f* on the axes provided.

23. 1991 AB-5

Let f be a function that is even and continuous on the closed interval [-3, 3]. The function f and its derivatives have the properties indicated in the table below.

x	0	$0<x<1$	1	$1<x<2$	2	$2<x<3$
$f(x)$	1	Positive	0	Negative	-1	Negative
$f'(x)$	Undefined	Negative	0	Negative	Undefined	Positive
$f''(x)$	Undefined	Positive	0	Negative	Undefined	Negative

(a) Find the x-coordinates of each point at which f attains an absolute maximum value or an absolute minimum value. For each x-coordinate you give, state whether f attains an absolute maximum or an absolute minimum.

(b) Find the x-coordinate of each point of inflection on the graph of f. Justify your answer.

(c) In the xy-plane provided below, sketch the graph of a function with all the given characteristics of f.

24. 1997 AB-4

Let f be the function given by $f(x) = x^3 - 6x^2 + p$, where p is an arbitrary constant.

(a) Write an expression for $f'(x)$ and use it to find the relative maximum and minimum values of f in terms of p. Show the analysis that leads to your conclusion.

(b) For what values of the constant p does f have 3 distinct real roots?

(c) Find the value of p such that the average value of f over the closed interval [-1, 2] is 1.

25. 1999 AB-4

Suppose that the function f has a continuous second derivative for all x, and that $f(0) = 2, f'(0) = -3$, and $f''(0) = 0$. Let g be a function whose derivative is given by $g'(x) = e^{-2x}(3f(x) + 2f'(x))$ for all x.

(a) Write an equation of the line tangent to the graph of f at the point where $x = 0$.

(b) Is there sufficient information to determine whether or not the graph of f has a point of inflection when $x = 0$? Explain your answer.

(c) Given that $g(0) = 4$, write an equation of the line tangent to the graph of g at the point where $x = 0$.

(d) Show that $g''(x)=e^{-2x}(-6f(x)-f'(x)+2f''(x))$. Does g have a local maximum at $x = 0$? Justify your answer.

26. MC '98 AB 19

If $f''(x) = x(x + 1)(x - 2)^2$, then the graph of f has inflection points when $x =$

(A) -1 only (B) 2 only (C) -1 and 0 only (D) -1 and 2 only (E) -1, 0, and 2 only

27. 1982 AB-6, BC-3

A tank with a rectangular base and rectangular sides is to be open at the top. It is to be constructed so that its width is 4 meters and its volume is 36 cubic meters. If building the tank costs \$10 per square meter for the base and \$5 per square meter for the sides, what is the cost of the least expensive tank?

28. MC '88 AB 45

The volume of a cylindrical tin can with a top and a bottom is to be 16π cubic inches. If a minimum amount of tin is to be used to construct the can, what must be the height, in inches, of the can?

(A) $2\sqrt[3]{2}$ (B) $2\sqrt{2}$ (C) $2\sqrt[3]{4}$ (D) 4 (E) 8

29. MC '98 AB 90

If the base b of a triangle is increasing at a rate of 3 inches per minute while its height h is decreasing at a rate of 3 inches per minute, which of the following must be true about the area A of the triangle?

(A) A is always increasing.
(B) A is always decreasing.
(C) A is decreasing only when $b < h$.
(D) A is decreasing only when $b > h$.
(E) A remains constant.

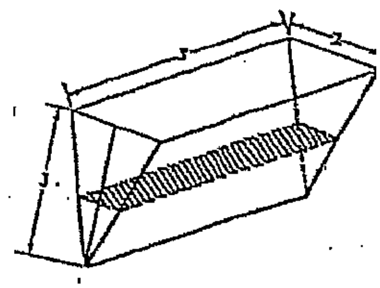
30. 1987 AB-5

The trough shown below is 5 feet long, and its vertical cross sections are inverted isosceles triangles with base 2 feet and height 3 feet. Water is being siphoned out of the trough at the rate of 2 cubic feet per minute. At any time t, let h be the depth and V be the volume of water in the trough.

(a) Find the volume of water in the trough when it is full.

(b) What is the rate of change in h at the instant when the trough is $\frac{1}{4}$ full by volume?

(c) What is the rate of change in the area of the surface of the water (shaded in the figure) at the instant when the trough is $\frac{1}{4}$ full by volume?



31. 2004 AB-3

A particle moves along the y-axis so that its velocity v at time $t \geq 0$ is given by $v(tO = 1 - \tan^{-1}(e^t)$. At time $t = 0$, the particle is at $y = -1$. (Note: $\tan^{-1}x = \arctan x$)

(a) Find the acceleration of the particle at time $t = 2$.

(b) Is the speed of the particle increasing or decreasing at time $t = 2$? Give a reason for your answer.

(c) Find the time $t \geq 0$ at which the particle reaches its highest point. Justify your answer.

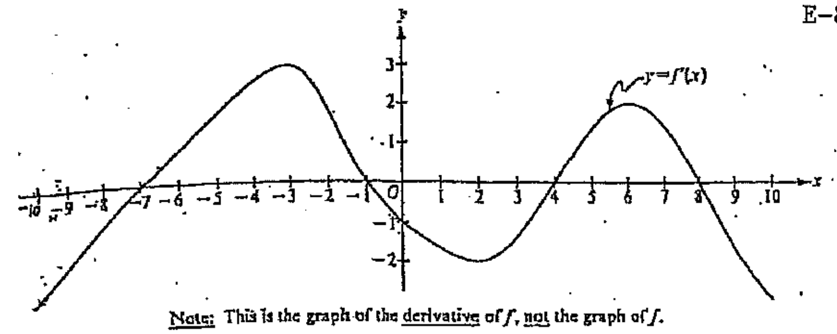
(d) Find the position of the particle at time $t = 2$. Is the particle moving toward the origin or away from the origin at time $t = 2$? Justify your answer.

32. MC '03 BC 82

The rate of change of the altitude of a hot-air balloon is given by $r(t) = t^3 - 4t^2 + 6$ for $0 \leq t \leq 8$. Which of the following expressions given the change in altitude of the balloon during the time altitude is decreasing?

(A) $\int_{1.572}^{3.514} r(t)dt$
(B) $\int_0^8 r(t)dt$
(C) $\int_0^{2.667} r(t)dt$
(D) $\int_{1.572}^{3.514} r'(t)dt$
(E) $\int_0^{2.667} r'(t)dt$

33. 1989 AB-5



The figure above shows the graph of f', the derivative of a function f. The domain of f is the set of all real numbers x such that $-10 \leq x \leq 10$.

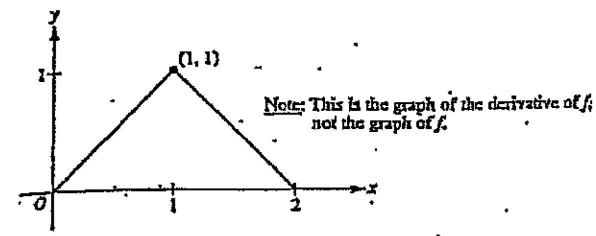
(a) For what values of x does the graph of f have a horizontal tangent?

(b) For what values of x in the interval (-10, 10) does f have a relative maximum? Justify your answer.

(c) For what values of x is the graph of f concave downward?

34. 1993 AB-5

The figure above shows the graph of f', the derivative of a function f. The domain of f is the set of all x such that $0 < x < 2$.



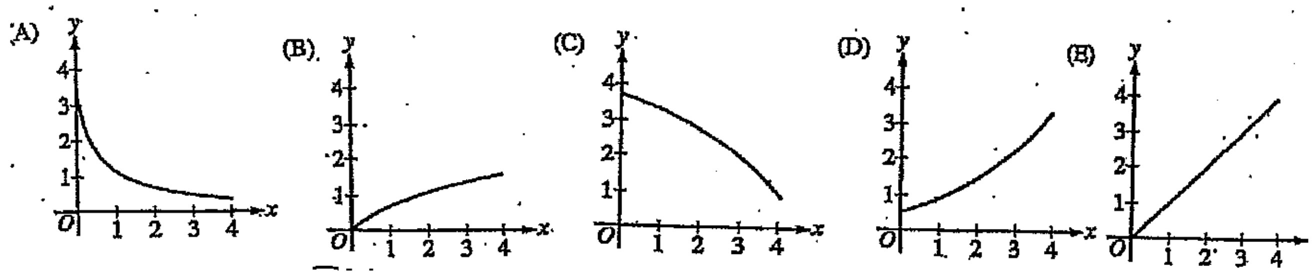
(a) Write an expression for $f'(x)$ in terms of x.

(b) Given that $f(1) = 0$, write an expression for $f(x)$ in terms of x.

(c) In the xy-plane provided below, sketch the graph of $y = f(x)$.

35. MC '03 BC 85

If a trapezoidal sum overapproximates $\int_0^4 f(x)dx$, and a right Riemann sum underapproximates $\int_0^4 f(x)dx$, which of the following could be the graph of $y = f(x)$?



36. 1984 AB-3, BC-1

Let R be the region enclosed by the X-axis, the Y-axis, the line $x = 2$, and the curve $y = 2e^x + 3x$.

(a) Find the area of R by setting up and evaluating a definite integral. Your work must include an antiderivative.

(b) Find the volume of the solid generated by revolving R about the Y-axis by setting up and evaluating a definite integral. Your work must include an antiderivative.

37. MC '93 BC 41

Let $f(x) = \int_{-2}^{x^2-3x} e^{t^2} dt$. At what value of x is $f(x)$ a minimum?

(A) For no value of x (B) $\frac{1}{2}$ (C) $\frac{3}{2}$ (D) 2 (E) 3

38. MC '98 AB 82

If f is a continuous function and if $F'(x) = f(x)$ for all real numbers x, then $\int_1^3 f(2x)dx =$

(A) $2F(3) - 2F(1)$ (C) $2F(6) - 2F(2)$ (E) $\frac{1}{2}F(6) - \frac{1}{2}F(2)$

(B) $\frac{1}{2}F(3) - \frac{1}{2}F(1)$ (D) $F(6) - F(2)$

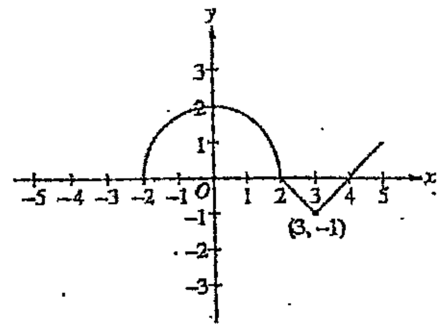
39 1976 AB-6

(a) Given $5x^3 + 40 = \int_c^x f(t)dt$.

(i) Find $f(x)$.

(ii) Find the value of c.

(b) If $F(x) = \int_x^3 \sqrt{1 + t^{16}}dt$, find $F'(x)$.



40. 1997 AB-5, BC-5

The graph of a function f consists of a semicircle and two line segments as shown above. Let g be the function given by $g(x) = \int_0^x f(t)dt$.

(a) Find $g(3)$.

(b) Find all values of x on the open interval (-2, 5) at which g has a relative maximum. Justify your answer.

(c) Write an equation for the line tangent to the graph of g at $x = 3$.

(d) Find the x-coordinate of each point of inflection of the graph of g on the open interval (-2, 5). Justify your answer.

41. 1981 AB-7

Let f be a continuous function that is defined for all real numbers x and that has the following properties.

(i) $\int_1^3 f(x)dx = \frac{5}{2}$

(ii) $\int_1^5 f(x)dx = 10$

(a) Find the average (mean) value of f over the closed interval [1, 3].

(b) Find the value of $\int_3^5 [2f(x) + 6]dx$.

(c) Given that $f(x) = ax + b$, find the values of a and b.

42. MC '03 AB 23

$\frac{d}{dx}\left(\int_0^{x^2} \sin(t^3)\, dt\right) =$

(A) $-\cos(x^6)$ (B) $\sin(x^3)$ (C) $\sin(x^6)$ (D) $2x\sin(x^3)$ (E) $2x\sin(x^6)$

43. MC '88 AB 43

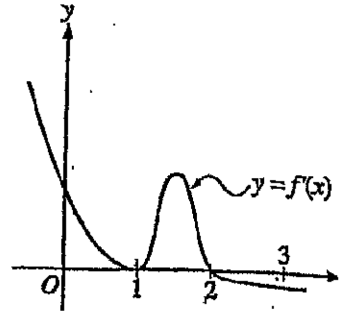
The volume of the solid obtained by revolving the region enclosed by the ellipse $x^2 + 9y^2 = 9$ about the x-axis is

(A) 2π (B) 4π (C) 6π (D) 9π (E) 12π

44. MC '93 BC 39

If $\frac{dy}{dx} = \frac{1}{x}$, then the average rate of change of y with respect to x on the closed interval [1, 4] is

(A) $-\frac{1}{4}$ (B) $\frac{1}{2}\ln 2$ (C) $\frac{2}{3}\ln 2$ (D) $\frac{2}{5}$ (E) 2



45. MC '03 BC 90

The graph of f', the derivative of the function f, is shown above. If $f(0) = 0$, which of the following must be true?

I. $f(0) > f(1)$
II. $f(2) > f(1)$
III. $f(1) > f(3)$

(A) I only (B) II only (C) III only
(D) I and II only (E) II and III only

46. 2004: AB/BC-1

Traffic flow is defined as the rate at which cars pass through an intersection, measured in cars per minute. The traffic flow at a particular intersection is modeled by the function F defined by $F(t) = 82 + 4\sin\left(\frac{t}{2}\right)$ for $0 \le t \le 30$, where $F(t)$ is measured in cars per minute.

(a) To the nearest whole number, how many cars pass through the intersection over the 30-minute period?
(b) Is the traffic flow increasing or decreasing at $t = 7$? Give a reason for your answer.
(c) What is the average value of the traffic flow over the time interval $10 \le t \le 15$? Indicate units of measure.
(d) What is the average rate of change of the traffic flow over the time interval $10 \le t \le 15$? Indicate units of measure.

47. MC '98 BC 91

t(sec)	0	2	4	6
$a(t)$ (ft/sec^2)	5	2	8	3

The data for the acceleration $a(t)$ of a car from 0 to 6 seconds are given in the table above. If the velocity at $t = 0$ is 11 feet per second, the approximate value of the velocity at $t = 6$, computed using a left-hand Riemann sum with three subintervals of equal length, is

(A) 26 ft/sec (B) 30 ft/sec (C) 37 ft/sec (D) 39 ft/sec (E) 41 ft/sec

48. MC '73 AB 28

A point moves in a straight line so that its distance at time t from a fixed point on the line is $8t - 3t^2$. What is the *total* distance covered by the point between $t = 1$ and $t = 2$?

(A) 1 (B) $\frac{4}{3}$ (C) $\frac{5}{3}$ (D) 2 (E) 5

49. 1997: AB-1

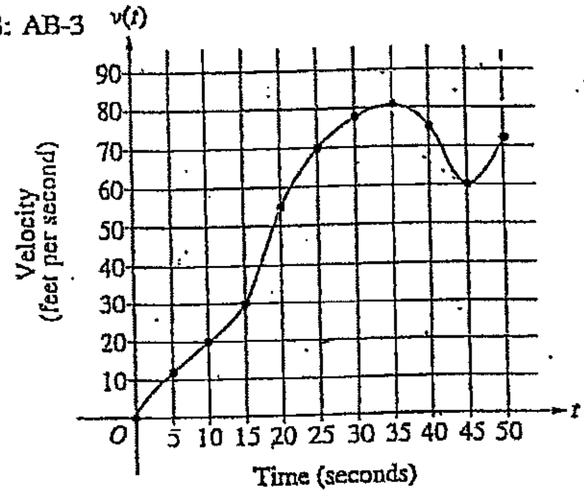
A particle moves along the *x*-axis so that its velocity at any time $t \geq 0$ is given by $v(t) = 3t^2 - 2t - 1$. The position *x(t)* is 5 for *t* = 2.

(a) Write a polynomial expression for the position of the particle at any time $t \geq 0$.

(b) For what values of *t*, $0 \leq t \leq 3$, is the particle's instantaneous velocity the same as its average velocity on the closed interval [0, 3]?

(c) Find the total distance traveled by the particle from time *t* = 0 until time *t* = 3.

50. 1998: AB-3



t (seconds)	*v(t)* (feet per second)
0	0
5	12
10	20
15	30
20	55
25	70
30	78
35	81
40	75
45	60
50	72

The graph of the velocity *v(t)*, in ft/sec. of a car traveling on a straight road for $0 \leq t \leq 50$ is shown above. A table of values for *v(t)*, at 5 second intervals of time *t* is shown to the right of the graph.

(a) During what intervals of time is the acceleration of the car positive? Give a reason for your answer.

(b) Find the average acceleration of the car, in ft/sec^2, over the interval $0 \leq t \leq 50$.

(c) Find one approximation for the acceleration of the car, in ft/sec^2 ,at *t* = 40. Show the computations you used to arrive at your answer.

(d) Approximate $\int_0^{50} v(t)dt$ with a Riemann sum, using the midpoints of five subintervals of equal length. Using correct units, explain the meaning of this integral.

51. 1996: AB-2

Let *R* be the region in the first quadrant under the graph of $y = \frac{1}{\sqrt{x}}$ for $4 \leq x \leq 9$.

(a) Find the area of *R*.

(b) If the line *x* = *k* divides the region *R* into two regions of equal area, what is the value of *k*?

(c) Find the volume of the solid whose base is the region *R* and whose cross sections cut by planes perpendicular to the *x*-axis are squares.

52. 1981: BC-6

(a) A solid is constructed so that it has a circular base of radius *r* centimeters and every plane section perpendicular to a certain diameter of the base is a square, with a side of the square being a chord of the circle. Find the volume of the solid.

(b) If the solid described in part (a) expands so that the radius of the base increases at a constant rate of $\frac{1}{2}$ centimeters per minute, how fast is the volume changing when the radius is 4 centimeters?

53. MC '93 AB 33

If $\frac{dy}{dx} = 2y^2$ and if $y = -1$ when $x = 1$, then when $x = 2$, $y =$

(A) $-\frac{2}{3}$ (B) $-\frac{1}{3}$ (C) 0 (D) $\frac{1}{3}$ (E) $\frac{2}{3}$

54. MC '93 AB 42

A puppy weighs 2.0 pounds at birth and 3.5 pounds two months later. If the weight of the puppy during its first 6 months is increasing at a rate proportional to its weight, then how much will the puppy weigh when it is 3 months old?

(A) 4.2 pounds (B) 4.6 pounds (C) 4.8 pounds (D) 5.6 pounds (E) 6.5 pounds

55. 1989: AB-6

Oil is being pumped continuously from a certain oil well at a rate proportional to the amount of oil left in the well; that is, $\frac{dy}{dt} = ky$, where y is the amount of oil left in the well at any time t. Initially there were 1,000,000 gallons of oil in the well, and 6 years later there were 500,000 gallons remaining. It will no longer be profitable to pump oil when there are fewer than 50,000 gallons remaining.

(a) Write an equation for y, the amount of oil remaining in the well at any time t.

(b) At what rate is the amount of oil in the well decreasing when there are 600,000 gallons of oil remaining?

(c) In order not to lose money, at what time t should oil no longer be pumped from the well?

56. 1992: AB-6

At time $t, t \geq 0$, the volume of a sphere is increasing at a rate proportional to the reciprocal of its radius. At $t = 0$, the radius of the sphere is 1 and at $t = 15$, the radius is 2. (The volume V of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.)

(a) Find the radius of the sphere as a function of t.

(b) At what time t will the volume of the sphere be 27 times its volume at $t = 0$?

57. 1999: AB-1

A particle moves along the y-axis with velocity given by $v(t) = t\sin(t^2)$ for $t \geq 0$.

(a) In which direction (up or down) is the particle moving at time $t = 1.5$? Why?

(b) Find the acceleration of the particle at time $t = 1.5$. Is the velocity of the particle increasing at $t = 1.5$? Why or why not?

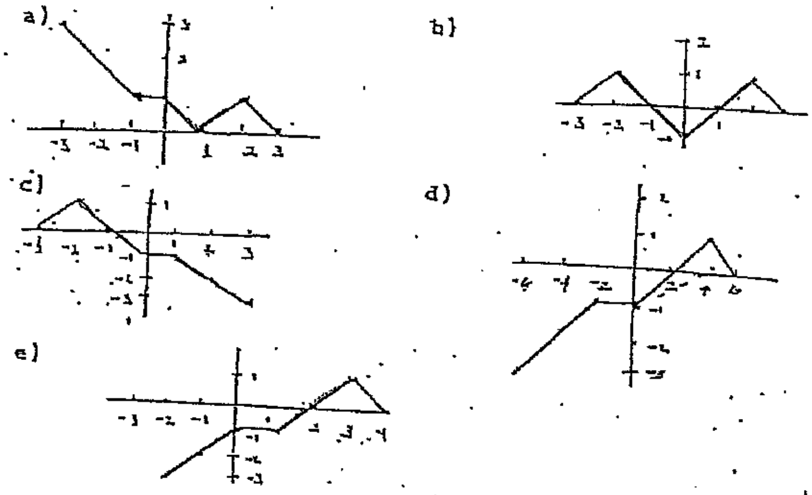
(c) Given that $y(t)$ is the position of the particle at time t and that $y(0) = 3$, find $y(2)$.

(d) Find the total distance traveled by the particle from $t = 0$ to $t = 2$.

CHAPTER E ANSWERS

% Correct	#	Answer
1	1.	E
26	2.	C
75	3.	E
62	4.	A

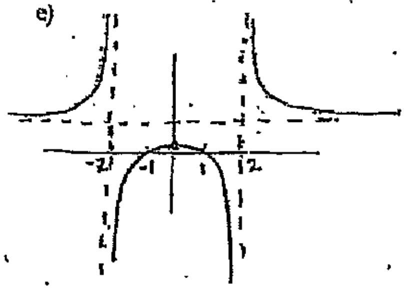
5.



6. (a) ¼ (b) $x = \pm 1$ (c) $x = \pm 2$, vertical asymptote $y = 1$, horizontal asymptote

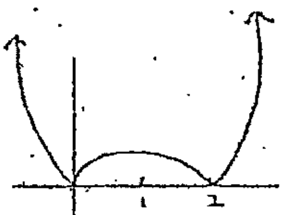
(d) x- axis symmetry

e)

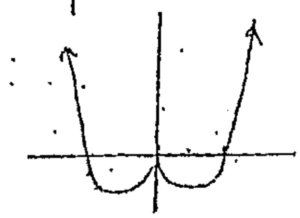


% Correct	#	Answer
48	7.	E
67	8.	A
13	9.	C
	10.	D
66	11.	D
91	12.	B
33	13.	D

14. a)



c)



(b) $\lim_{h\to 0^+}\left(\frac{2h-h^2}{h}\right)=2$

$\lim_{h\to 0^-}\left(\frac{h^2-2h}{h}\right)=-2$

Derivative does not exist at $x=0$

(d) (i) $f(|x|)=0$

(ii) $\lim_{h\to 0}\left(|x|^2-2|x|\right)=0$

Continuous at $x=0$

15. (a) $f'(2)+g'(2)=6+7=13$

(b) $f(2)\cdot g'(2)+f'(2)\cdot g(2)=14+6\pi$

(c) $\frac{g(2)\cdot f'(2)-f(2)\cdot g'(2)}{\left(g(2)\right)^2}=\frac{6\pi-14}{\pi^2}$

(d) $h'(x)=f'\left(g(x)\right)\cdot g'(x)$

$h'(1)=f'\left(g(1)\right)\cdot g'(1)=f'(2)\cdot 4=24$

(e) $y=g'(x)$ $\quad x=g(y)$

$\frac{dy}{dx}=\frac{1}{g'(y)}$ $\quad \frac{1}{g'(1)}=\frac{1}{4}$

16. (a) $k=\frac{9}{2}$ (b) $k=\frac{3w}{2}$ (c) 21 units/sec (d) $-\frac{7}{100}$ units^2/sec, decreasing

% Correct	#	Answer
40	17.	D
42	18.	B

19. (a) Show (b) $y=2$ (c) $\frac{2}{7}$, maximum

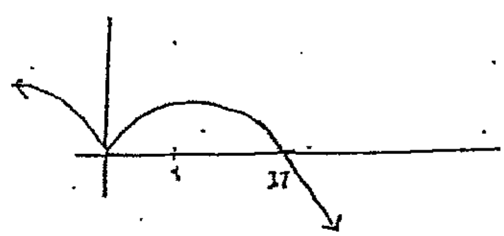
% Correct	#	Answer
3	20.	D

21. (a) $\frac{dy}{dx}=\frac{16x-5y}{5x+3y^2}$ (b) $y=3x-13$ (c) $k\approx -0.4$

(d) $-8(4.2)^2+5k(4.2)+k^3=-149$ (e) $k\approx -.373$

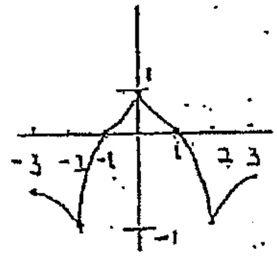
22. (a) (0, 8) (b) (8, 16) (c) (0, 0) (d) when $x\neq 0$

(e)



23. (a) max: $x = 0$ min: $x = 2, x = -2$ (b) $x = 1, x = -1$

(c)



24. (a) $f' = 3x^2 - 12x$ Rel max: $f(0) = p$ Rel min: $f(4) = p - 32$

(b) $0 < p < 32$ (c) $p = 5.75$

25. (a) $-3x + 2$ (b) No, no way to verify that f'' changes sign about $x = 0$

(c) $y = 4$ (d) Yes, at $x = 0$

% Correct	#	Answer
28	26.	C

27. $330

% Correct	#	Answer
43	28.	D
36	29.	D

30. (a) 15 ft^3 (b) - 2/5 ft/min (c) - 4/3 ft^2/min

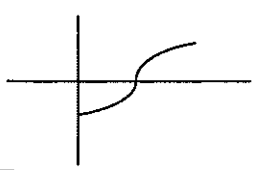
31. (a) - 0.132 or – 0.133 (b) Speed increases $a(2) < 0$ and $v(2) < 0$ (c) $t = 0.443$

% Correct	#	Answer
54	32.	A

33. (a) $x = -7, -1, 4, 8$ (b) $x = -1,\ x = 8$ (c) $(-3, 2)$; $(6, 10)$

34. (a) $\begin{cases} x \text{ if } 0 < x \leq 1 \\ 2 - x \text{ if } [1, 2] \end{cases}$ (b) $\begin{cases} \frac{x^2}{2} - \frac{1}{2} & (0, 1] \\ 2x - \frac{x^2}{2} - \frac{3}{2} & [1, 2) \end{cases}$

(c)



% Correct	#	Answer
69	35.	A

36. (a) $2e^2 + 4$ (b) $4\pi e^2 + 20\pi$

% Correct	#	Answer
32	37.	C
24	38.	E

39. (a) i) $f(x) = 15x^2$ ii) -2
(b) $f'(x) = -\sqrt{1+x^{16}}$

40. (a) $\pi - \frac{1}{2}$ (b) $x = 2$ (c) $y - \left(\pi - \frac{1}{2}\right) = -x + 3$ (d) $x = 0, x = 3$

41. (a) 5/4 (b) 27 (c) $a = 5/4$, $b = -5/4$

% Correct	#	Answer
29	42.	E
24	43.	B
52	44.	C
59	45.	B

46. (a) 2474 cars (b) decreasing (c) 81.899 cars/min. (d) 1.517 or 1.518 cars/min.^{2}

% Correct	#	Answer
26	47.	E
10	48.	C

49. (a) $x(t) = t^3 - t^2 - t + 3$ (b) $t = \frac{2+\sqrt{76}}{6} \approx 1.786$ (c) 17

50. (a) $0 < t < 35$ or $45 < t < 50$ (b) $\frac{36}{25}$ ft/sec^2
(c) $a(40) \approx -2.1$ ft/sec^2 or -1.2 ft/sec^2 or -3 ft/sec^2
(d) 2530 ft. Distance traveled in feet from $t = 0$ to 50 sec.

51. (a) 2 (b) 25/4 (c) $\ln\frac{9}{4} \approx .811$

52. (a) $\frac{16r^3}{3}$ (b) $\approx$ 19.680 billion gals/yr. (c) $\approx$ 27.668

% Correct	#	Answer
14	53.	B
30	54.	B

55. (a) $y = 10^6 e^{\frac{t}{6}\ln\frac{1}{2}}$ (b) 100,000 ln 2 gal/yr. (c) $t \geq (6 \ln 20)/\ln 2$

56. (a) $r = (1+t)^{\frac{1}{4}}$ (b) $t = 80$

57. (a) up, $v(1.5) > 0$ (b) ≈ -2.049; No, velocity is decreasing since $v'(1.5) < 0$
(c) $y(2) \approx 3.827$ (d) ≈ 1.173

Simple Things Students Can Do To Improve Their AP Calculus Exam Score

This paper proposes some important things to consider in preparation for the AP Calculus exam and gives examples of the "tough" questions from recent AP exams as illustrations of common misunderstanding among students. Certainly the most important point for student preparation for the AP exam is to cover all of the curriculum as mandated by the AP course syllabus. Hopefully this curriculum coverage can be accomplished with time for exam question review available during the last two to four weeks prior to the AP exam. This exam review time allows the student the chance to see questions which stretch across the year and which creatively combine concepts from various chapters in their text. It is not unusual to hear a student ask why this question was not in their text—it probably was there. It's just that the students get so used to their own author's style that hearing a slightly different phrasing of a question makes them believe they have never done this before. It is important that during this time the student does the work, discussions and presentations, because the time just before the AP exam is a time for the student to believe he/she can make a passing grade or, better than that, a score of **5**.

The teacher becomes the observer, coach, advisor, and cheerleader. As much as is possible the teacher needs to watch for common class errors and search for more problem examples to practice. It is also quite important that the teacher grade work using the AP rubrics where they are available. And if it is at all possible to hold AP test practice sessions, it is during these sessions that the student gets the feel of the test structure and the time allotments and the inherent pressure of the exam experience. This type of practice gives a psychological advantage on the **real day** as students know what they are up against, having lived through it in practice.

And now for a list of things to improve:

1. Learn to read the problem, carefully highlight key words, and be sure you answered the question.
   Key words: Closed interval or open interval –Be sure your answer stays in the domain.
   Did it ask for an optimal (max or min) function value? And you gave the domain value at which it optimizes.
   Name the coordinates of a point.
   Justify your answer.—Make clear mathematical arguments, don't give calculator reasons.

2. Know the vocabulary of single variable Calculus – See the attached AP compendium of terms, pages J-1 & J-2.

3. Draw and/or use geometry to reinforce your thoughts. Look at these AP exam questions:

'85, AB 27 25% correct ANS: D

$\int_0^3 |x-1|\,dx =$

(A) 0 (B) $\frac{3}{2}$ (C) 2 (D) $\frac{5}{2}$ (E) 6

'85, AB 38 25% correct ANS: C

Let *f* and *g* have continuous first and second derivatives everywhere. If $f(x) \le g(x)$ for all real *x*, which of the following must be true?

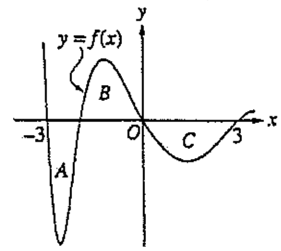
I. $f'(x) \le g'(x)$ for all real x

II. $f''(x) \le g''(x)$ for all real x

III. $\int_0^1 f(x)dx \le \int_0^1 g(x)dx$

(A) None (B) I only (C) III only (D) I and II only (E) I, II, and III

'03, AB 77 23% correct ANS: C



The regions A, B, and C in the figure above are bounded by the graph of the function f and the x-axis. If the area of each region is 2, what is the value of $\int_{-3}^{3}(f(x)+1)dx$?

(A) -2 (B) -1 (C) 4 (D) 7 (E) 12

4 a). Check the answers for reasonableness.

'69, AB 26 7% correct ANS: C

$\int_{0}^{1}\sqrt{x^2-2x+1}\,dx$ is

(A) -1 (B) $-\frac{1}{2}$ (C) $\frac{1}{2}$ (D) 1

(E) none of the above

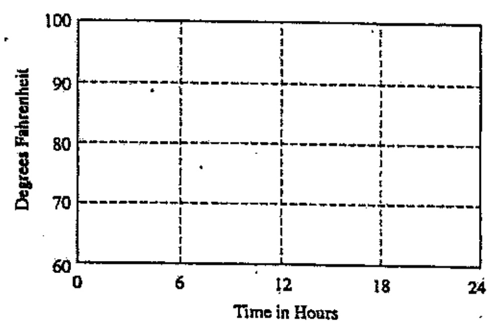
4 b). Use dimensional analysis to support the units your answers call for.

98 AB/BC 5 Free Response

The temperature outside a house during a 24-hour period is given by

$F(t)=80-10\cos\left(\frac{\pi t}{12}\right), 0\le t\le 24$, where

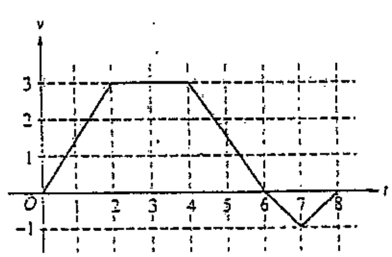
$F(t)$ is measured in degrees Fahrenheit and t is measured in hours.



(a) Sketch the graph of F on the grid.
(b) Find the average temperature, to the nearest degree Fahrenheit, between $t = 6$ and $t = 14$.
(c) An air conditioner cooled the house whenever the outside temperature was at or above 78 degrees Fahrenheit. For what values of t was the air conditioner cooling the house?
(d) The cost of cooling the house accumulates at the rate of \$0.05 per hour for each degree the outside temperature exceeds 78 degrees Fahrenheit. What was the total cost, to the nearest cent, to cool the house for this 24-hour period?

4 c). Distinguish between the terms, displacement versus total distance and velocity versus speed.

'97, AB 9 27% correct ANS: B



A bug begins to crawl up a vertical wire at time $t = 0$. The velocity v of the bug at time t, $0\le t\le 8$, is given by the function whose graph is shown above.
What is the total distance the bug traveled from $t = 0$ to $t = 8$?

(A) 14 (B) 13 (C) 11 (D) 8 (E) 6

4 d). If you give answers in decimal form, give them with a minimal 3-decimal place accuracy. Be careful of intermediate rounding errors when working through a problem.

5. Know the BIG 7 THEOREMS of single-variable Calculus.

- **Intermediate Value Theorem**: Suppose f is continuous on the closed interval [a, b] and let k be any number between $f(a)$ and $f(b)$, where $f(a) \neq f(b)$, then there exists a number c in (a, b) such that $f(c) = k$
- If a function $f(x)$ is differentiable at $x = a$, then $f(x)$ is continuous at $x = a$.
- **Endpoint Extrema Theorem**: Every continuous function on a closed interval must have a maximum and a minimum.
- **Rolle's Theorem**: If function $f(x)$ is continuous on a closed interval $[a, b]$, differentiable on the open interval (a, b), and if $f(a) = f(b)$, then $f'(c) = 0$ for at least one number c in (a, b).
- **Mean Value Theorem for Derivatives**: If function $f(x)$ is continuous on a closed interval $[a, b]$ and is differentiable on the open interval (a, b), then there exists at least one number c in (a, b) such that $f'(c) = \dfrac{f(b) - f(a)}{b - a}$.
- **Fundamental Theorem of Calculus**: Suppose f is continuous on $[a, b]$.

  i) If $g(x) = \int_a^x f(t)\,dt$, then $g'(x) = f(x)$.

  ii) $\int_a^b f(x)dx = F(b) - F(a)$, where F is any antiderivative of f, that is, $F' = f$.

  or a "reform" version of the Fundamental Theorem:

  Total Change Theorem: The definite integral of a rate of change is the total change: $\int_a^b F'(x)dx = F(b) - F(a)$.
- **Mean-Value Theorem for Definite Integrals**: If f is continuous on $[a, b]$, then there exists a number c in [a, b] such that $\int_a^b f(x)dx = f(c)(b - a)$.

Routinely AP questions will require the student to recognize and use appropriate theorem justifications. A recent example is the 2007 AB-3 Free-Response, a question with an all-time statistical, low national mean of 0.95 out of a possible 9 points. See this question on page C-18, problem 2.

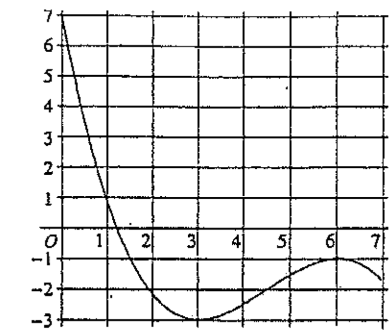
6. Understand the test design, practice with test timelines at least once, use an AP grading rubric to score your free response answers. Attack the <u>hardest</u> questions on each test, make them <u>simple</u>! The following questions are the "statistically hardest" questions on the two most recently released tests for AB and BC in '98 and '03.

'03 AB 27 18% correct ANS: B

Let f be the function defined by $f(x) = x^3 + x$. If $g(x) = f^{-1}(x)$ and $g(2) = 1$, what is the value of $g'(2)$?

(A) $\frac{1}{13}$ (B) $\frac{1}{4}$ (C) $\frac{7}{4}$ (D) 4 (E) 13

'03, BC 18 24% correct ANS: C

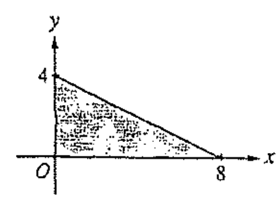


Graph of f

The graph of the function f shown in the figure above has horizontal tangents at $x = 3$ and $x = 6$. If $g(x) = \int_0^{2x} f(t)\,dt$, what is the value of $g'(3)$?

(A) 0 (B) -1 (C) -2 (D) -3 (E) -6

'98, AB 86 19% correct ANS: C



The base of a solid is a region in the first quadrant bounded by the x-axis, the y-axis, and the line $x + 2y = 8$, as shown in the figure above. If the cross sections of the solid perpendicular to the x-axis are semicircles, what is the volume of the solid?

(A) 12.566 (B) 14.661 (C) 16.755 (D) 67.021 (E) 134.041

'98, BC 26 20% correct ANS: E

The population $P(t)$ of a species satisfies the logistic differential equation $\frac{dP}{dt} = P\left(2 - \frac{P}{5000}\right)$, where the initial population $P(0) = 3{,}000$ and t is the time in years. What is $\lim_{t\to\infty} P(t)$?

(A) 2,500 (B) 3,000 (C) 4,200 (D) 5,000 (E) 10,000

7. Never give up! Keep your pencil moving the entire test. Don't expect to get <u>everything</u> right and don't stall on a question, just move on and come back if time permits. On the multiple choice there is a penalty for wrong answers, so you should attempt to answer all questions. On the free response questions show as much work as you possibly can, even if you cannot reach a final answer. It's the conceptual work that will earn you the most points. Final answers can be left in unsimplified numerical form if you are pushed for time.