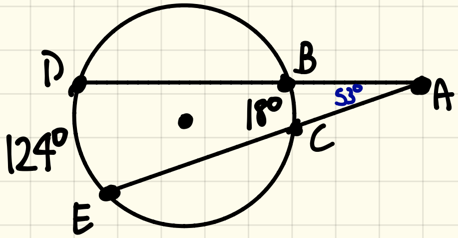
3.5 Arc Length & Area of a Sector

Old Find Arc Measures in Circles

① Find m< BAC.



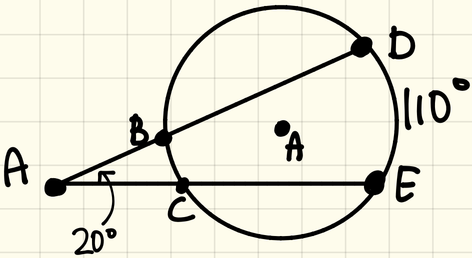
$$\frac{\text{Big Arc} - \text{Small Arc}}{2} = \text{Outside Angle}$$

$$\frac{\overset{\frown}{DE} - \overset{\frown}{BC}}{2} = \angle BAC$$

$$\frac{124° - 18°}{2} = \angle BAC$$

$$53° = \angle BAC$$

② Find $\overset{\frown}{BC}$.



$$\frac{\text{Big Arc} - \text{Small Arc}}{2} = \text{Outside Angle}$$

$$\frac{\overset{\frown}{DE} - \overset{\frown}{BC}}{2} = \angle BAC$$

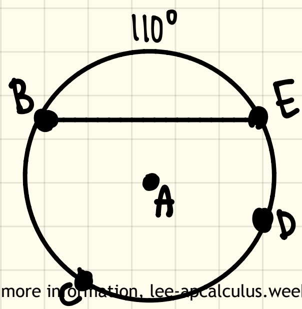
$$\frac{110° - \overset{\frown}{BC}}{2} = 20°$$

$$110° - \overset{\frown}{BC} = 40°$$

$$-\overset{\frown}{BC} = -70°$$

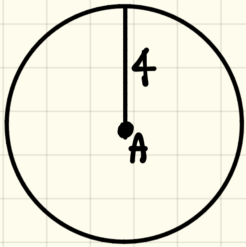
$$\overset{\frown}{BC} = 70°$$

- Arc Measure is the angle that an arc makes at the center of the circle of which it is part.



[new] Arc Length & Area of Sector

Let's consider the circle with a radius of 4. Find the circumference of the circle.



Circumference = 2π radius

$C = 2\pi r$

$C = 2\pi(4)$

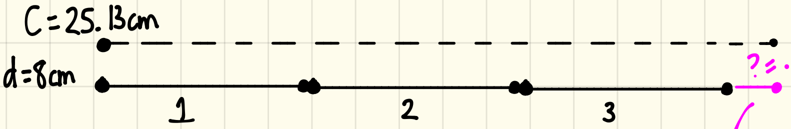
$= 8\pi \approx 25.13$

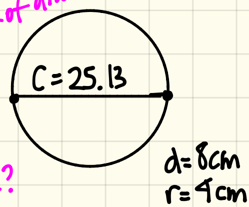
exact answer — approximation or rounded answer

What does the circumference of a circle mean?

Circumference means the distance (length) around a circle.

Let's prove the formula of a circumference. Let's "dissect" the circle by unrolling the circle.





How much is the small space? $\approx .14$

diameter = 6cm

d = 6cm * 3.14 = 25.13

diameter * π = Circumference

[Example] A circular flower garden has a radius of 3 feet. Find the circumference of the garden to the nearest hundredths.

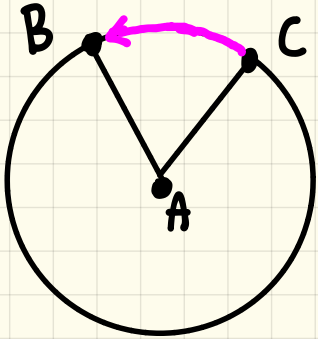
$C = 2\pi r$

$C = 2\pi(3)$

$= 6\pi \approx 18.85$ feet.

The distance around the circular flower garden is approximately 18.85 ft.

Arc Length The distance (length) along the curved line making the arc.



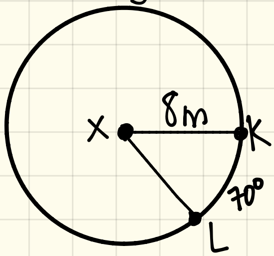
(NOT the same as arc measure ⇒ will NOT have a degree amount)

$$\boxed{\text{Arc Length} = \left(\frac{\text{arc measure}^\circ}{360^\circ}\right) 2\pi r}$$

$$\text{Length of BC} = \left(\frac{m\overset{\frown}{BC}}{2}\right) 2\pi(\overline{AB}).$$

[Examples] Find the arc lengths.

① Find length of $\overset{\frown}{KL}$.



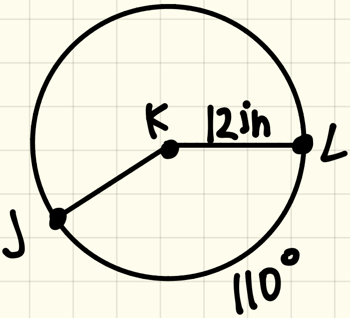
$$\text{Arc Length} = \left(\frac{\text{arc measure}^\circ}{360^\circ}\right) 2\pi r$$

$$\text{length of } \overset{\frown}{KL} = \left(\frac{m\overset{\frown}{KL}}{360^\circ}\right) 2\pi(\overline{XK})$$

$$\text{length of } \overset{\frown}{KL} = \left(\frac{70^\circ}{360^\circ}\right) 2\pi(8)$$

$$= \frac{28\pi}{9} \approx 9.77\text{m}$$

② Find length of $\overset{\frown}{JL}$.



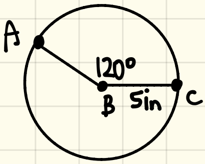
$$\text{Arc Length} = \left(\frac{\text{arc measure}°}{360°}\right) 2\pi r$$

$$\text{length of } \overset{\frown}{JL} = \left(\frac{m\overset{\frown}{JL}}{360°}\right) 2\pi r$$

$$\text{length of } \overset{\frown}{JL} = \left(\frac{110°}{360°}\right) 2\pi(12)$$

$$= \frac{22\pi}{3} \approx 23.04 \text{ in}$$

③ Find length of $\overset{\frown}{AC}$.



$$\text{Arc Length} = \left(\frac{\text{arc measure}°}{360°}\right) 2\pi r$$

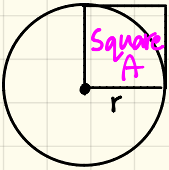
$$\text{Length of } \overset{\frown}{AC} = \left(\frac{m\overset{\frown}{AC}}{360°}\right) (2\pi r)$$

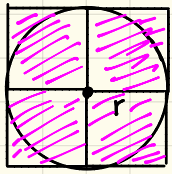
$$= \left(\frac{120°}{360°}\right) 2\pi(5)$$

$$= \frac{10\pi}{3} \approx 10.47 \text{ in}$$

$m\overset{\frown}{AC} = m\angle ABC$
because of
$m\angle ABC$ is the central angle.

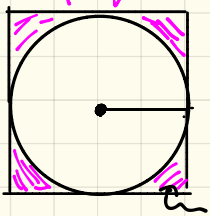
Let's consider a circle with radius, r.
How do we find the amount of space fill in a circle?





Area of Square A $= r * r = r^2$

Area of 4 squares $= r^2 * 4$

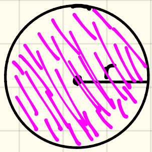


but we don't want the space in the corner.
(We need to multiply by something less than 4)

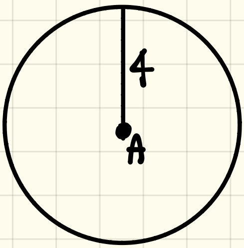
Area of 4 squares $= r^2 * 4$

$= r^2 * \underline{3.14}$

$= r^2 \pi$



Let's consider the circle with a radius of 4. Find the area of the circle.



Area $= \pi(\text{radius})^2$

$A = \pi r^2$

$A = \pi(4)^2$

$A = 16\pi \approx 50.27$

exact answer — approximation or rounded answer

[Example] A circular flower garden has a radius of 3 feet. Find the area of the garden to the nearest hundredths.

$A = \pi r^2$

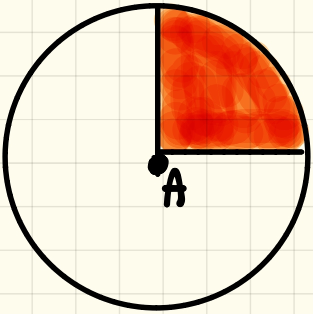
$A = \pi(3)^2$

$A = 9\pi \approx 28.27 \text{ ft}^2$

The space filled in the circular flower garden is approximately 28.27 ft^2.

Area of a Sector

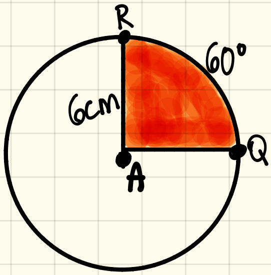
The amount of space bounded by 2 radii of the circle & their intercepted arc.



$$\text{Area of Sector} = \left(\frac{\text{arc measure}^\circ}{360^\circ}\right)\pi r^2$$

[Examples] Find the area of the sector.

1. Find area of $\overset{\frown}{RQ}$



$$\text{Area of Sector} = \left(\frac{\text{arc measure}^\circ}{360^\circ}\right)\pi r^2$$

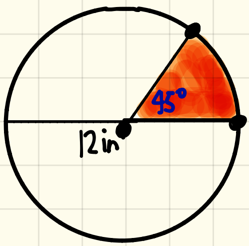
$$\text{Area of } \overset{\frown}{RQ} = \left(\frac{m\overset{\frown}{RQ}}{360^\circ}\right)\pi(\overline{RA})^2$$

$$= \left(\frac{60^\circ}{360^\circ}\right)\pi(6)^2$$

$$= 6\pi \approx 18.85\text{cm}^2$$

② Find the area of a sector with central angle of $45°$ if the diameter of circle is 12 inches.

Draw picture:



if diameter = 12in, then radius = 6 in.

$$\text{Area of Sector} = \left(\frac{\text{arc measure}°}{360°}\right)\pi r^2$$

$$\text{Area of Sector} = \left(\frac{45°}{360°}\right)\pi(6)^2$$

$$= \frac{9\pi}{2} \simeq 14.14 \text{ in}^2$$