

## 3.5 Trigonometric Derivatives

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Standard:

MCD1e

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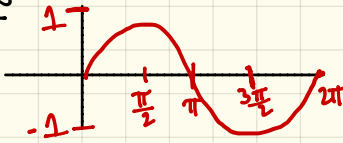
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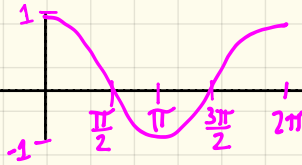
# Old Trig Review

## Trig Functions

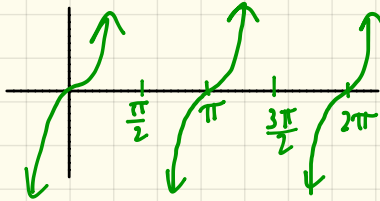
•  $\sin(x)$



•  $\cos(x)$



•  $\tan(x)$



## Reciprocal Functions

•  $\csc(x) = \frac{1}{\sin x}$

•  $\sec(x) = \frac{1}{\cos x}$

•  $\cot(x) = \frac{1}{\tan(x)}$

## Trig Value Chart — Memorize

Degrees	Radians	$\sin(x)$	$\cos(x)$
0	0, $\pi$	0	1
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
45°	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
90°	$\frac{\pi}{2}$	1	0

[Examples] Evaluate Trig Functions.

$$\textcircled{1} \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$$

$$\textcircled{2} \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

$$\begin{aligned}\textcircled{3} \sin\left(\frac{5\pi}{4}\right) &= \\ &= \sin(225) \\ &= -\sin(45) \\ &= -\frac{\sqrt{2}}{2}.\end{aligned}$$

# new Trig Derivatives

Let's consider  $f(x) = \sin(x)$ . Find the derivative.

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\sin(x+h) - \sin(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\sin x \cosh + \cos x \sinh - \sin x}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\sin x \cosh - \sin x + \cos x \sinh}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\sin x (\cosh - 1) + \cos x \sinh}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\sin x (\cosh - 1)}{h} + \lim_{h \rightarrow 0} \frac{\cos x \sinh}{h}$$

$$= \lim_{h \rightarrow 0} \sin x \cdot \lim_{h \rightarrow 0} \frac{\cosh - 1}{h} + \lim_{h \rightarrow 0} \cos x \cdot \lim_{h \rightarrow 0} \frac{\sinh}{h}$$

$$= \sin(x) \cdot \frac{\cos(0) - 1}{1} + \cos x \cdot 1 \text{ (graphically)}$$

$$= (\sin x \cdot 0) + (\cos x \cdot 1)$$

$$= \cos x$$



Side note: Double Angle Formula

$$\sin(x+h) = \sin x \cosh + \cos x \sinh$$

# Trig Derivatives - Memorize

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\cot x) = -\csc^2 x$$

[Examples] Find the derivatives of each function.

$$PR = f \cdot g' + g \cdot f'$$

$$\textcircled{1} f(x) = x^3 \sin x$$

$$\begin{aligned} f'(x) &= (x^3)(\cos x) + (\sin x)(3x^2) \\ &= x^3 \cos x + 3x^2 \sin x \\ &= x^2(x \cos x + 3 \sin x). \end{aligned}$$

$$\textcircled{3} f(x) = x \tan x$$

$$\begin{aligned} f'(x) &= (x)(\sec^2 x) + (\tan x)(1) \\ &= x \sec^2 x + \tan x. \end{aligned}$$

$$\textcircled{5} f(x) = \cos x \sin x$$

$$\begin{aligned} f'(x) &= (\cos x)(\cos x) + (\sin x)(-\sin x) \\ &= \cos^2 x - \sin^2 x \end{aligned}$$

$$\textcircled{7} f(x) = \sqrt{x} + \sec x$$

$$f'(x) = \frac{1}{2\sqrt{x}} + \sec x \tan x$$

$$\textcircled{2} f(x) = \frac{x^3}{\cos x}$$

$$QR = \frac{g \cdot f' - f \cdot g'}{g^2}$$

$$f'(x) = \frac{(\cos x)(3x^2) - (x^3)(-\sin x)}{(\cos x)^2}$$

$$= \frac{3x^2 \cos x + x^3 \sin x}{\cos^2 x}$$

$$= \frac{x^2(3 \cos x + x \sin x)}{\cos^2 x}$$

$$\textcircled{6} f(x) = x^3 + \csc x$$

$$f'(x) = 3x^2 - \csc x \cot x$$

$$\textcircled{8} f(x) = 6 \cot x$$

$$f'(x) = -6 \csc^2 x.$$

$$\textcircled{9} f(x) = \frac{x^2 \sin x}{\cos x}$$

$$f'(x) = \frac{(\cos x) [(x^2)(\cos x) + (\sin x)(2x)] - (x^2 \sin x)(-\sin x)}{(\cos x)^2}$$

$$= \frac{\cos x [x^2 \cos x + 2x \sin x] + x^2 \sin^2 x}{\cos^2 x}$$

$$= \frac{x^2 \cos^2 x + 2x \cos x \sin x + x^2 \sin^2 x}{\cos^2 x}$$

$$= \frac{x^2 \cancel{\cos^2 x}}{\cancel{\cos^2 x}} + \frac{2x \cancel{\cos x} \sin x}{\cancel{\cos^2 x}} + \frac{x^2 \sin^2 x}{\cos^2 x}$$

$$= x^2 + \frac{2 \sin x}{\cos x} + \frac{x^2 \sin^2 x}{\cos^2 x}.$$