4.1 Arithmetic Sequences

Standards:

F.BF.2

F.LE.2

F.LE.3

$\boxed{\text{Old}}$ Patterns

Determine the pattern in each situation.

1. 12, 24, 36, 48, ... - Adding 12
2. -12, -13, -14, -15, ... - Subtracting 1
3. 4, 16, 64, 256, 1024, ... - Multiply 4
4. 4, 8, 12, 16, 20, 24, ... - Adding 4

$\boxed{\text{new}}$ Sequences

A sequence is a string of numbers that contains a certain pattern.

(For Example) 12, 24, 36, 48, ... is a sequence because the string of numbers holds a pattern of "Adding 12".

Notation of a Sequences:

a_n — where $\underline{n}$ is referring to the term number in the sequence.

(Example) 12, 24, 36, 48, ...

$$\begin{array}{cccc} 12 & 24 & 36 & 48 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ a_1 & a_2 & a_3 & a_4 \\ (1^{st}\#) & (2^{nd}\#) & (3^{rd}\#) & (4^{th}\#) \end{array}$$

FACT The values in the range are called the term of the sequence.

Domain: $1 \quad 2 \quad 3 \quad 4 \quad 5 \quad \cdots \quad n$
(position in the sequence)

Range: $a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5 \quad \cdots \quad a_n$
(the actual sequence)

(Example) For the sequence 12, 24, 36, 48, ..., label the domain & range.

Domain	1	2	3	4	...
Range	12	24	36	48	...

More new Arithmetic Sequences

Arithmetic Sequence — is formed by adding (or subtracting) a particular value each time to the value just before it.

We notate arithmetic sequences with d meaning "common differences".

[Examples]

① 4, 8, 12, 16, 20, 24, ... $d = 4$

② −1, 1, 3, 5, ... $d = 2$

③ −2, −4, −6, −8, ... $d = -2$

[Examples 2] Find a_3, a_5 & a_7.

① 4, 8, 12, 16, ... $- a_3 = 12 \quad a_5 = 20, \ a_7 = 28$

② -1, 1, 3, 5, ... $- a_3 = 3, \ a_5 = 7, \ a_7 = 11$

③ -2, -4, -6, -8, ... $a_3 = -6, \ a_5 = -10, \ a_7 = -14$

There are 2 ways to express a sequence:

① Recursive

② Explicit

$\boxed{1}$ Recursive Formula for Arithmetic Sequences

$$a_1 = \boxed{}, \quad a_n = a_{n-1} + d$$

a_1 → 1st term

a_n → "Now Term"

a_{n-1} → "Before Term"

d → Common Difference

Note: All you have to do is identify the a_1 term & the common difference.

[Example] Write the recursive formula for the following arithmetic sequences.

① 4, 8, 12, 16, ... $a_1 = 4, \ a_n = a_{n-1} + 4$

② -1, 1, 3, 5, ... $a_1 = -1, \ a_n = a_{n-1} + 2$

③ -2, -4, -6, -8, ... $a_1 = -2, \ a_n = a_{n-1} - 2$

Explicit Formula for Arithmetic Sequences

Let's consider $a_n = 2n - 3$. Create the Domain/Range Chart from this rule.

Domain	1	2	3	4	...
Range	-1	1	3	5	...

How do we go backwards & get this formula back?

Domain	0	1	2	3	4	...
Range	-3	-1	1	3	5	...

$+2 \quad +2 \quad +2$

$d = 2$

$a_n = 2n - 3$

Explicit Formula: $a_n = dn + a_0$

$\boxed{\text{Note:}}$ You need to identify the "d" & a_0 (go backwards to get 0^{th} term.)

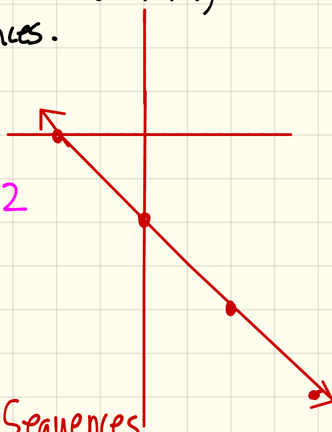
[Examples] Find the explicit formula for the sequences.

① $-4, -6, -8, -10, \ldots$ (Also graph #1)

Domain	0	1	2	3	4	...
Range	-2	-4	-6	-8	-10	...

$-2 \quad -2 \quad -2$

$a_n = -2n - 2$



$\boxed{\text{please note:}}$ Arithmetic Sequences model linear functions.

② 12, 15, 18, 21, ...

Domain	0	1	2	3	4	...
Range	8	12	15	18	21	...

$12 \xrightarrow{+4} 15 \xrightarrow{+4} 18$

$a_n = 4n + 8$

③ 18, 24, 30, 36, ...

Domain	0	1	2	3	4	...
Range	12	18	24	30	36	...

$18 \xrightarrow{+6} 24 \xrightarrow{+6} 30 \xrightarrow{+6} 36$

$a_n = 6n + 12$

[Example] Find a_{50} for the following sequences.

① 50, 60, 70, 80, ...

Domain	0	1	2	3	4	...
Range	40	50	60	70	80	...

$50 \xrightarrow{+10} 60 \xrightarrow{+10} 70 \xrightarrow{+10} 80$

$a_n = 10n + 40$

$$a_{50} = 10(50) + 40$$
$$= 500 + 40$$
$$= 540.$$

② $-14, -17, -20, -23, \ldots$

Domain	0	1	2	3	4	...
Range	-11	-14	-17	-20	-23	...

$-3 \quad -3 \quad -3$

$a_n = -3n - 11$

$a_{50} = -3(50) - 11$
$= -180 - 11$
$= -169$

[Examples] Write the recursive formula in explicit form.

① $a_1 = 4, \quad a_n = a_{n-1} - 6$

The sequence is $4, -2, -8, -14, \ldots$

Domain	0	1	2	3	4	...
Range	10	4	-2	-8	-14	...

$\boxed{a_n = -6n + 10}$

② $a_1 = -3, \quad a_n = a_{n-1} + 10$

The sequence is $-3, 7, 17, 27, \ldots$

Domain	0	1	2	3	4	...
Range	-13	-3	7	17	27	...

$+10 \quad +10 \quad +10$

$a_n = 10n - 13$

[Example] Use the explicit formula to find the recursive formula.

① $a_n = 2n - 3$

$a_1 = 2(1) - 3$
$= 2 - 3$
$= -1$

$a_1 = \boxed{}$, $a_n = a_{n-1} + d$

$a_1 = -1$, $a_n = a_{n-1} + 2$

② $a_n = -6n + 10$

$a_1 = -6(1) + 10$
$= 4.$

$a_1 = 4$, $a_n = a_{n-1} - 6.$