

4.2 Writing Equations of Parabolas

Old Graphing Parabolas, Focus & Directrix

① Find the focus & directrix for $x = -\frac{1}{8}(y-4)^2 - 2$

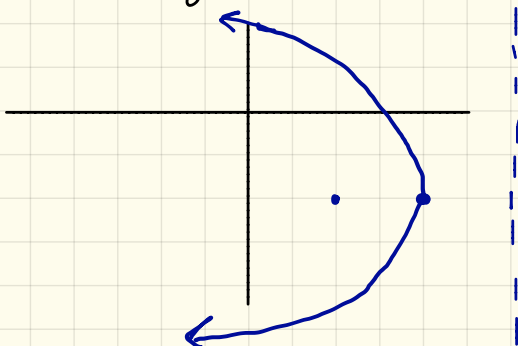
vertex: $(4, -2)$

shrink by a factor of $\frac{1}{8}$

$$a = -\frac{1}{8}$$

$$p = \frac{1}{4(-\frac{1}{8})} = -2$$

Focus: $(4 + (-2), -2) = (2, -2)$
Directrix: $x = 4 - (-2) = 6$

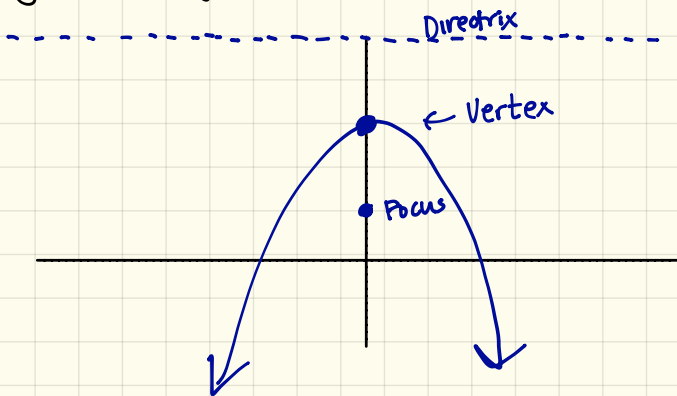


② Find the vertex when given the focus & directrix.

Focus: $(0, 1)$

Directrix: $y = 5$

vertex: $(0, 3)$



[note:] It's always good to sketch! It helps identify either focus/directrix or vertex in certain situations once you identify "p".

New Writing Equations of Parabolas

Let's recall:

- Formula for p : $\frac{1}{4a}$
- vertex form for Parabolas

$$y = a(x-h)^2 + k$$

vertex: (h, k)

Focus: $(h, k+p)$
Directrix: $y = k-p$

$$x = a(y-k)^2 + h$$

vertex: (k, h)

Focus: $(k+p, h)$
Directrix: $y = k-p$

Our goal: write the equation of a parabola when only given focus & directrix of a parabola.

[Examples] Find the equation of the parabola when given focus & directrix.

① Focus: $(0, 3)$
Directrix: $y = 5$

need to find a :

$$p = \frac{1}{4a}$$

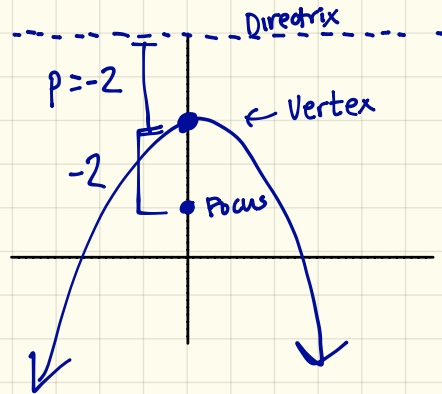
$$-2 = \frac{1}{4a}$$

$$-8a = 1$$
$$a = -\frac{1}{8}$$

vertex: $(0, 3)$

$$y = a(x-h)^2 + k$$
$$y = -\frac{1}{8}(x-0)^2 + 3$$

$$y = -\frac{1}{8}x^2 + 3$$



② Focus: $(2, -5)$; Directrix: $x = -2$

need to find a : vertex: $(.5, -5)$

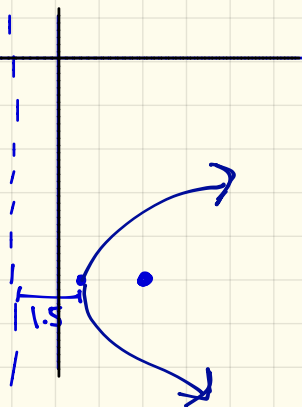
$$p = \frac{-1}{4a}$$

$$x = a(x-k)^2 + h$$

$$x = \frac{1}{6} \left(x - \frac{1}{2}\right)^2 - 5$$

$$1.5 = \frac{-1}{4a}$$

$$6a = 1$$
$$a = \frac{1}{6}$$



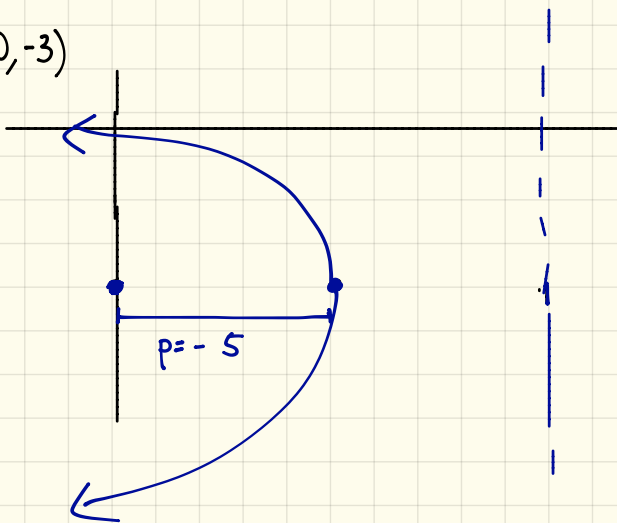
③ Vertex: $(5, -3)$, Focus: $(0, -3)$

need to find a :

$$p = \frac{1}{4a}$$

$$5 = \frac{1}{4a}$$

$$20a = 1$$
$$a = \frac{1}{20}$$



Vertex: $(5, -3)$

$$x = a(y-k)^2 + h$$
$$x = \frac{1}{20} (y-5)^2 - 3$$