4.3 Minimum & Maximum Values

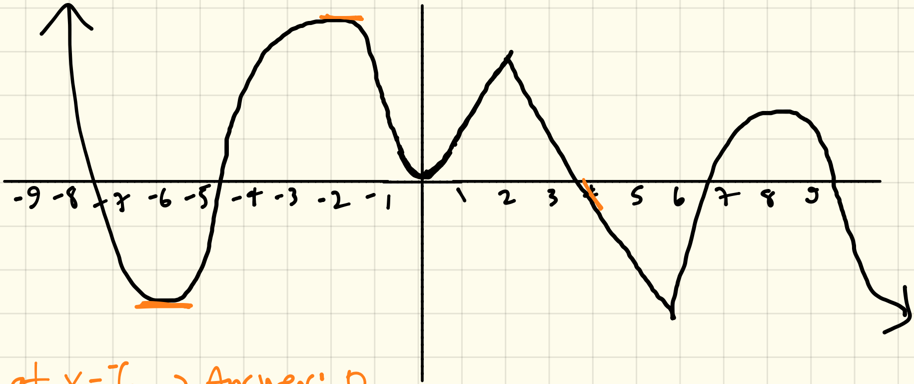
Standards:

MCA3

MCA3b

01d Interpretation of Derivatives

For the following graphs, let's estimate the slopes of tangent lines at certain points:



(a) at $x = -6$ → Answer: 0
(b) at $x = 4$ → Answer: -1

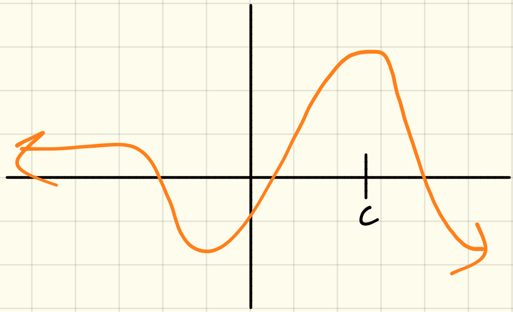
(c) at $x = -2$ → Answer: 0
(d) at $x = 7$ → Answer: D.N.E (sharp corner)

$\boxed{\text{Conclusion}}$ At maximum & minimum values the slope of the tangent line is 0.

New: Maximum & Minimum Values

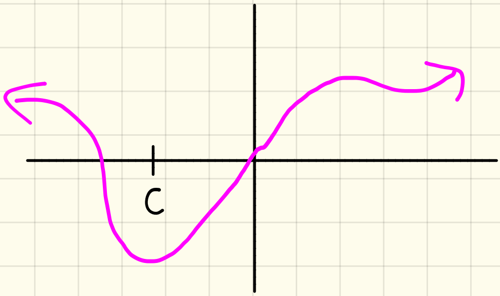
Absolute Maximum: The largest y-value of a function attained among all the x-values in the domain of a function.

An abs max occurs at $x=c$ if $f(c) \geq f(x)$ for all x's in the domain of $f(x)$.



Absolute Minimum: The lowest y-value a function attains among the x-values in the domain of a function.

An abs min occurs at $x=c$ if $f(c) \leq f(x)$ for all x's in the domain of $f(x)$.



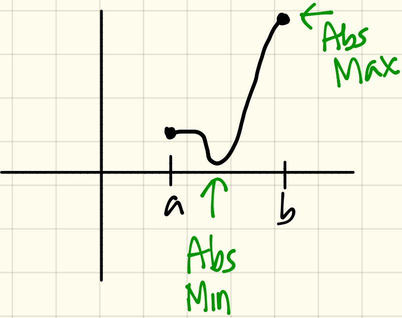
Note: The Absolute min & max values are called the <u>EXTREME VALUES</u> of a function.

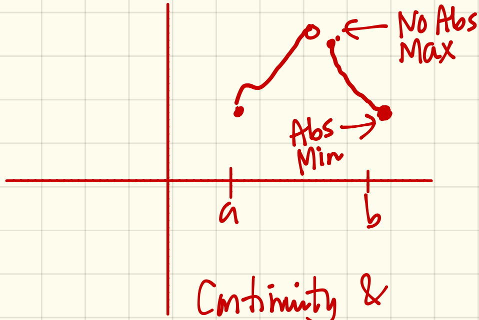
Also note: Local min & max values are "tops" & "bottoms" of hills of a graph, but <u>not</u> necessarily the absolute highest or lowest points.

The Extreme Value Theorem

If $f(x)$ is continuous on the closed interval $[a,b]$, then $f(x)$ attains its maximum & minimum values somewhere in $[a,b]$

Illustration:



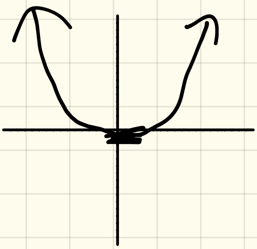


Continuity & Closed intervals Matter!

Fermat's Theorem

If $f(x)$ has a local max (or minimum) at $x=c$ & $f'(c)$ exists, then $f'(c)=0$.

Let's consider $f(x)=x^2$.



$$f(x)=x^2$$
$$f'(x)=2x$$
$$f'(x)=2x=0$$
$$x=0.$$

There exists a max or min value at $x=0$.

A critical number of a function is a value (c) in the domain of $f(x)$ such that either:

① $f'(c)=0$ or ② $f'(c) \rightarrow$ does not exist.

Our Goal: A method for finding absolute maximum & minimum values for a continuous function on a closed interval.

A Closed Interval Method — A way to locate the abs max & min values of a function on a closed interval

Step 1: Find the critical numbers of $f(x)$ on $[a,b]$.
→ need to take derivative, set $f'(x)$ equal to zero & solve for x.

Step 2: Evaluate $f(c)$ for all c's from Step 1. c = critical #'s

Step 3: Evaluate $f(a)$ & $f(b)$ separately.

Step 4: The largest value from step 2 and step 3 is the abs max. The smallest value is the abs min.

[Example 1] Find the abs max & abs min of $f(x) = 3x^4 - 4x^3$ on the closed interval $[-1, 2]$.

Step 1:

$$f(x) = 3x^4 - 4x^3$$
$$f'(x) = 12x^3 - 12x^2 = 0$$
$$12x^2(x-1) = 0$$
$$x = 0, 1$$

Step 2/3

critical #'s:

$f(0) = 0$
$f(1) = -1$

endpoints:

$f(-1) = 7$
$f(2) = 16$

Step 4

The absolute maximum is 16 occuring at $x=2$ and the absolute minimum is -1 occuring at $x=1$.

[Example 2] Find the abs max & min of $f(x) = (x^2-1)^3$ on $[-1, 2]$.

Step 1

$$f(x) = (x^2-1)^3$$
$$f'(x) = 3(x^2-1)^2(2x)$$
$$= 6x(x^2-1)^2$$

$$f'(x) = 6x(x^2-1)^2 = 0$$
$$= 6x(x^2-1)(x^2-1)$$
$$= 6x(x-1)(x+1)(x-1)(x+1)$$
$$x = 0, 1, -1$$

Step 2/3

critical #'s

$f(0) = 0$
$f(1) = 0$
$f(-1) = -1$

endpoints:

$f(-1) = -1$
$f(2) = 27$

Step 4

The abs max is 27 occuring at $x=2$ and the abs min is -1 occuring at $x=-1$.