4.4 Systems of Circles Parabolas & Lines

Let's recall what system of equations means. Systems of Equations is a simultaneous set of lines (or curves) meeting at common point(s).

Let's consider the system of linear equations $\begin{cases} y = 2x + 1 \\ y = -2x - 3 \end{cases}$

What are the different methods one can use to solve systems of linear equations?

↙ Let's use

① Graphing   ② Substitution   ③ Elimination

at least of the variable in one eqtn must be solved for.

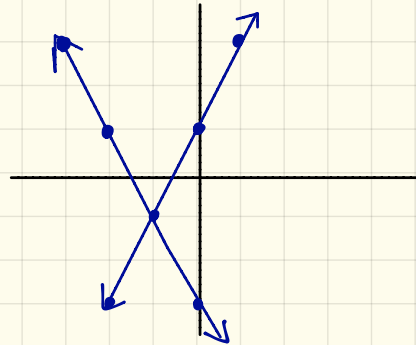Solve for the common point using substitution method.

$\begin{cases} y = 2x + 1 \\ y = -2x - 3 \end{cases}$

Graph the 2 lines to show system & common point.

$2x + 1 = -2x - 3$ get y: sub -1 for x
$+2x \quad = +2x$
_____
$4x + 1 = -3$
$\quad 1 = -1$
_____
$4x = -4$ $y = 2(-1) + 1$
$\overline{4} \quad \overline{4}$ $y = -2 + 1$
$x = -1$ $y = -1$

$(-1, -1)$

| New | System of Circles, Parabolas, and/or lines.

Let's consider the following graph:

Find the common points algebraically (Solve the system)

$$\begin{cases} x^2 + y^2 = 25 \\ y = 3x - 5 \end{cases}$$

$x^2 + (3x-5)^2 = 25$
$x^2 + (3x-5)(3x-5) = 25$
$x^2 + 9x^2 - 15x - 15x + 25 = 25$
$10x^2 - 30x + 25 = 25$
$\underline{\ -25 = -25}$
$10x^2 - 30x = 0$
$10x(x - 3) = 0$
$x = 0, 3$

A circle & a line.

$y = 3x - 5$

$x^2 + y^2 = 25$

Find the y's:

when x=0

$y = 3(0) - 5$
$y = -5$

When x=3

$y = 3(3) - 5$
$y = 4$

Common points are $(0, -5)$ and $(3, 4)$.
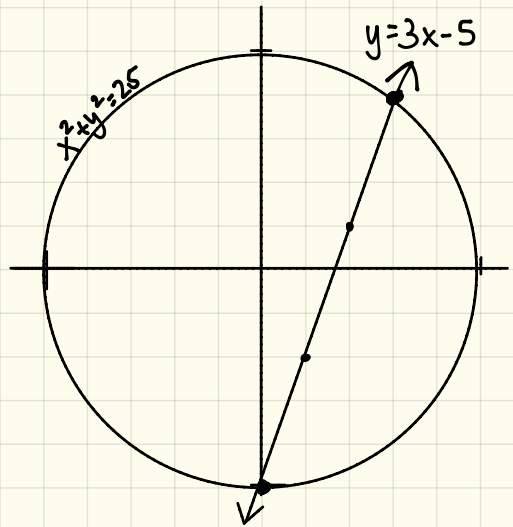
| note: | Use substitution method is usually the best & efficient method to use!

[Example 1] Solve the simultaneous equations $\begin{cases} x^2 + y^2 = 10 \\ y = x + 2 \end{cases}$.

(A circle & a line)

$$x^2 + y^2 = 10$$
$$y = x + 2$$

$$x^2 + (x+2)^2 = 10$$
$$x^2 + (x+2)(x+2) = 10$$
$$x^2 + x^2 + 2x + 2x + 4 = 10$$
$$2x^2 + 4x + 4 = 10$$
$$2x^2 + 4x - 6 = 0$$
$$2(x^2 + 2x - 3) = 0$$
$$2(x+3)(x-1) = 0$$
$$x = -3, \ x = 1$$

Find y:

when x = -3   when x = 1
$$y = (-3) + 2 \qquad y = (1) + 2$$
$$y = -1 \qquad\qquad y = 3$$

$$= (-3, -1) \text{ and } (1, 3)$$

[Example 3] Solve the system when a parabola & line intersect
$$\begin{cases} y = x^2 + 4x - 1 \\ y = 7x + 9 \end{cases}$$

$$x^2 + 4x - 1 = 7x + 9$$
$$x^2 - 3x - 10 = 0$$
$$(x-5)(x+2) = 0$$
$$x = 5, \ x = -2.$$

Find y:

When x = 5   When x = -2
$$y = 7(5) + 9 \qquad y = 7(-2) + 9$$
$$= 44 \qquad\qquad y = -5$$

Common points are (5, 44) and (-2, -5).