4.5 First Derivatives Test

Standards:

MCA3

MCA3b

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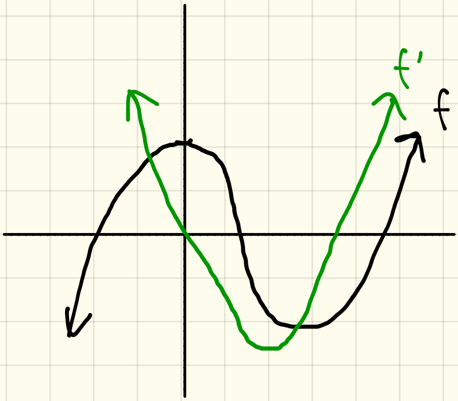
$\boxed{\text{old}}$ Sketching Derivatives

Find the derivative of the graph.



Conclusion:

- f is increasing, y-coordinates of f' are positive
- f is decreasing, y-coordinates of f' are negative.

$\boxed{\text{new}}$ First Derivatives Test

Let's consider the function: $f(x) = x^3 - x$.

Goal:

We want to figure out the characteristics of the graph of $x^3 - x$ by using the function ONLY!

(No Calculator!)

Recall: Increasing/Decreasing of Function

- If $f' > 0$ on an interval, then f is increasing.
- If $f' < 0$ on an interval, then f is decreasing.

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First Derivatives Test

Suppose c is a critical number of a continuous function $f(x)$.

- If f' changes sign from $+$ (positive) to $-$ (negative) at c, then f has a local maximum at c.
- If f' changes sign from $-$ to $+$ at c, then f has a local minimum at c.
- If f' doesn't change sign, then there is neither a local max or local min at c. (usually call these saddle points)

First Derivatives Test gives:

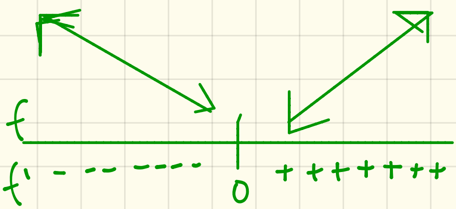
1. Increasing / Decreasing Intervals
2. Critical Points
3. Local Max / Local Min

[Example 1] $f(x) = x^2$

$f(x) = x^2$

$f'(x) = 2x = 0$

$x = 0$



Interval of Increase: $(0, \infty)$

Interval of Decrease: $(-\infty, 0)$

Local Min: $(0, f(0)) = (0, 0)$.

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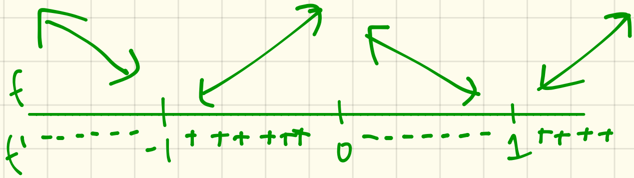
[Example 2] $f(x) = x^4 - 2x^2$

$f(x) = x^4 - 2x^2$

$f'(x) = 4x^3 - 4x = 0$

$4x(x^2 - 1) = 0$

$x = 0, -1, 1$



Intervals of Increase: $(-1, 0) \cup (1, \infty)$

Intervals of Decrease: $(-\infty, -1) \cup (0, 1)$

Local Maxs: $(0, f(0)) = (0, 0)$

Local Mins: $(-1, f(-1)) = (-1, -1)$

$(1, f(1)) = (1, -1)$

[Example 3] $f(x) = x^3 - x$

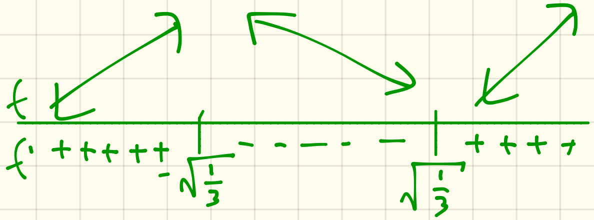
$f(x) = x^3 - x$

$f'(x) = 3x^2 - 1 = 0$

$3x^2 = 1$

$x^2 = \frac{1}{3}$

$x = \pm\sqrt{\frac{1}{3}}$



Intervals of Increase: $(-\infty, -\sqrt{\frac{1}{3}}) \cup (\sqrt{\frac{1}{3}}, \infty)$

Intervals of Decrease: $(-\sqrt{\frac{1}{3}}, \sqrt{\frac{1}{3}})$

Local Max: $(-\sqrt{\frac{1}{3}}, f(-\sqrt{\frac{1}{3}}))$ Local Min: $(\sqrt{\frac{1}{3}}, f(\sqrt{\frac{1}{3}}))$

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