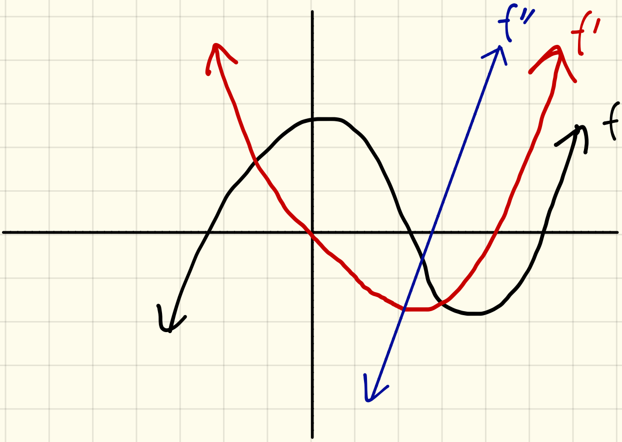
4.6 Second Derivatives Test

Standards:

MCA3

MCA3b

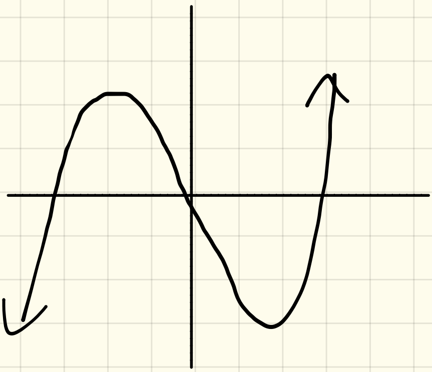
$\boxed{\text{Old}}$ Sketching Derivatives – sketch f' and f''



$\boxed{\text{new}}$ Second Derivatives Test

Let's consider the function: $f(x) = x^3 - x$.

Goal: The same as the First Derivatives Test.
To sketch graph using characteristics found by using function ONLY!



This was created by Keenan Xavier Lee, 2013. See my website for more information, lee-apcalculus.weebly.com.

The Second Derivatives gives:

1. concavity — local max/local mins
2. Inflection points

What is concavity?

- Just as the slope of the tangent line to the graph at the point $(x, f(x))$ describes the behavior of a function, concavity describes the behavior of the slope.

- As x increases (graph goes from left to right) on the following is true:
  - ↳ concavity is positive – so the slope is slowly increases

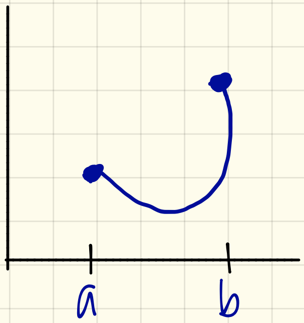
    "smile" concave up

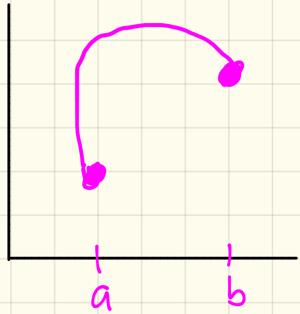
  - ↳ concavity is negative – so the slope is slowly decrease

    "frown" concave down

  - ↳ Concavity is equal to zero, so the slope is constant.

Let's consider these 2 cases:



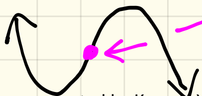


- f is increasing
- $f' > 0$, f' is increasing (getting steeper)
- $f'' > 0$

- f is decreasing
- $f' < 0$, f' is decreasing (less steep)
- $f'' < 0$

Concavity Test

- If $f'' > 0$ on an interval, then the graph of f is concave up  – "smiling"
- If $f'' < 0$ on an interval, then the graph of f is concave down  – "frowning"
- Points where the concavity changes are inflection points.



– point of inflection

[Example 1] $f(x) = x^4 - 2x^2$.

$f(x) = x^4 - 2x^2$

$f'(x) = 4x^3 - 4x$

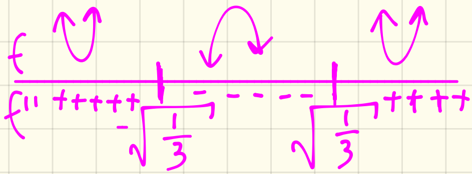
$f''(x) = 12x^2 - 4 = 0$

$4(3x^2 - 1) = 0$

$3x^2 - 1 = 0$

$x^2 = \frac{1}{3}$

$x = \pm\sqrt{\frac{1}{3}}$



Interval of Concave Up: $\left(-\infty, -\sqrt{\frac{1}{3}}\right) \cup \left(\sqrt{\frac{1}{3}}, \infty\right)$

Interval of Concave Down: $\left(-\sqrt{\frac{1}{3}}, \sqrt{\frac{1}{3}}\right)$

Points of Inflection: $\left(-\sqrt{\frac{1}{3}}, f\left(-\sqrt{\frac{1}{3}}\right)\right)$ and

$\left(\sqrt{\frac{1}{3}}, f\left(\sqrt{\frac{1}{3}}\right)\right)$