5.1 Average Value of Function

The average value of a function $f(x)$ on the interval $[a,b]$ is:

$$f_{ave} = \frac{1}{b-a} \int_a^b f(x)\, dx \, .$$

[Example 1] Find the average value of $f(x) = 4-x^2$ on $[0,3]$.

$$f_{ave} = \frac{1}{b-a} \int_a^b f(x)\, dx = \frac{1}{3-0} \int_0^3 4-x^2\, dx = \frac{1}{3} \int_0^3 4-x^2\, dx = \frac{1}{3}\left[4x - \frac{x^3}{3}\right]_0^3$$

$$= \frac{1}{3}\left[4(3) - \frac{(3)^3}{3}\right] - \left[4(0) - \frac{(0)^3}{3}\right] = \frac{1}{3}\left[12 - \frac{27}{3}\right] = \frac{1}{3}\left[12 - 9\right] = \frac{1}{3}\left[6\right] = \boxed{3}$$

[Example 2] Find the average value of $f(x) = \sin x$ on $[0, \pi]$.

$$f_{ave} = \frac{1}{b-a} \int_a^b f(x)\, dx = \frac{1}{\pi-0} \int_0^\pi \sin\pi\, dx = \frac{1}{\pi} \int_0^\pi \sin x\, dx = \frac{-1}{\pi}\left[\cos x\right]_0^\pi$$

$$= \frac{-1}{\pi}\left[\cos(\pi)\right] - \left[\cos(0)\right] = \frac{-1}{\pi}\left([-1] - [1]\right) = \frac{-1}{\pi}(-2) = \boxed{\frac{2}{\pi}}$$