5.1 Venn Diagrams, Overlapping Events & Mutually Exclusive Events

Old Simple Probability

Notation:
Probability of an event = P(event)

Formula:
$P(\text{event}) = \frac{\#\text{ of selected}}{\text{Total amount}}$

[Examples] Maria has a bag full of marbles — 4 red marbles, 8 grey marbles, 3 blue marbles, 5 yellow marbles.

① What's the probability that Maria picks a blue marble?

$P(\text{blue marble}) = \frac{3}{20} = 0.15$ (15%)

② What's the probability that Maria picks a grey marble?

$P(\text{grey marble}) = \frac{8}{20} = \frac{2}{5} = 0.4$ (40%)

③ What's the probability that Maria picks a red marble?

$P(\text{red marble}) = \frac{4}{20} = \frac{1}{5} = .2$ (20%)

new-A Sample Space

- Sample Space - the set of all possible outcomes for an event. "The universe".

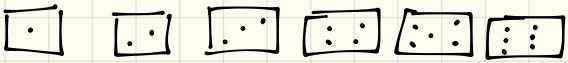
(Example) Let's consider a dice. What is the sample space for this object.

 → 6 possible outcomes

{1, 2, 3, 4, 5, 6} ← Sample space

- Subset - a part of the sample space (universe).

(Example) Let's consider a dice. What is the sample space for this object.

 → 6 possible outcomes

However, only take into consideration the even outcomes.

 ← 3 possibilities of the 6 possible outcomes

{2, 4, 6}

$\subset$ - symbol for subset.

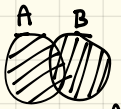
$\{2, 4, 6\} \subset \{1, 2, 3, 4, 5, 6\}$

"2, 4, 6 is a subset of 1, 2, 3, 4, 5, 6.

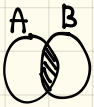
COMPOUND EVENTS - combines 2 or more events using the word "and" or the word "or".

Set Notation

- $A \cup B$ — "or" "union" — everything in both sets

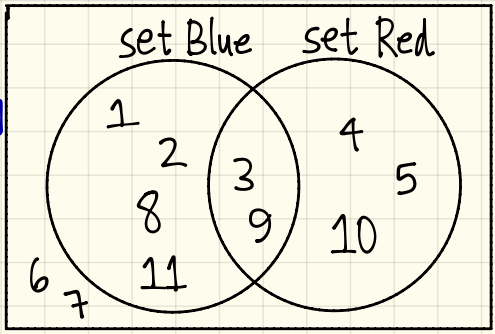


- $A \cap B$ — "and" "intersection" — only what is common in both sets



- A' — complement (or opposite) — everything NOT in set A

[Example] Consider set blue & set red.



① Blue $\cup$ Red = {1, 2, 8, 11, 4, 5, 10, 3, 9}

② Blue $\cap$ Red = {3, 9}

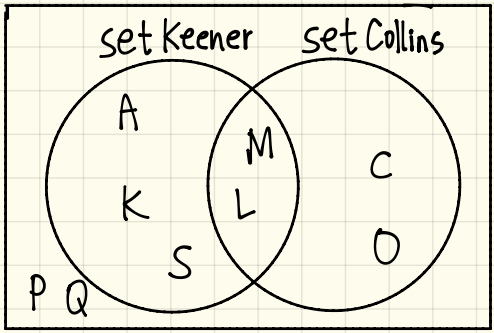
③ Blue' = {4, 5, 10, 6, 7}

④ (Blue $\cup$ Red)' = {6, 7}

⑤ (Blue $\cap$ Red)' = {1, 2, 8, 11, 4, 5, 10, 6, 7}

⑥ Red' = {1, 2, 3, 8, 11, 3, 9, 6, 7}

[Example] Consider set Keener & set Collins.



① Keener ∩ Collins = {M, L}

② Keener′ = {M, L, C, O, P, Q}

③ Keener ∪ Collins = {A, K, S, C, O, M, L}

④ Collins′ = {M, L, A, K, S, P, Q}

⑤ (Keener ∩ Collins)′ = {C, O, A, K, S, P, Q}

⑥ $P(\text{Keener}) = \frac{5}{9} = .55 = 55\%$

⑦ $P(\text{Collins}) = \frac{4}{9} = .\overline{44} = 44.4\%$

⑧ $P(\text{Keener}') = \frac{6}{9} = \overline{66.} = 66.6\%$

New-B Compound Probability

Compound Probability - the probability of 2 or more events using the word "and" or the word "or".

- "or" - ∪ → $P(A) \cup P(B) = P(A) + P(B)$
- "and" - ∩ → $P(A) \cap P(B) = P(A) * P(B)$

[Examples] Maria has a bag full of marbles — 4 red marbles, 8 grey marbles, 3 blue marbles, 5 yellow marbles.

① What's the probability Maria will get a blue marble or a grey marble?

$P(\text{blue}) \text{ or } P(\text{grey}) = P(\text{blue}) + P(\text{grey}) = \frac{3}{20} + \frac{8}{20} = \frac{11}{20} = .55$

55%

② What's the probability Maria will get a red marble and a yellow marble?

$P(\text{red}) \text{ and } P(\text{yellow}) = P(\text{red}) \cap P(\text{yellow}) = \frac{4}{20} * \frac{5}{20} = \frac{1}{20} = .05$

5%

③ What's the probability Maria will get 2 grey marbles?

$P(\text{grey}) \text{ and } P(\text{grey}) = \frac{8}{20} * \frac{8}{20} = \frac{4}{25} = .16$ 16%

④ What's the probability Maria will not get blue marbles?

$P(\text{blue}') = \frac{4+8+5}{20} = \frac{17}{20} = .85$ 85%

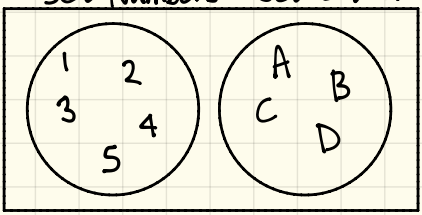
⑤ What's the probability Maria will orange marbles?

$P(\text{orange}) = \frac{0}{20} = 0\%$

new-C Mutually Exclusive & Overlapping

- Mutually Exclusive – 2 or more simultaneous events that have no common outcomes.

(Example)



No common outcomes!

Let's consider a deck of cards.
52 total cards, 4 suits, 13 cards in each suit.

What is the probability that Mya gets a King or gets a Ace?

→ Mutually Exclusive = no common outcomes

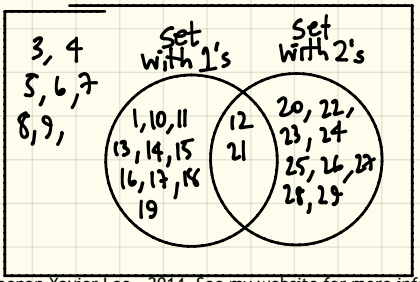
Subtract out the case(s) to not to consider it twice!

$$P(\text{King}) \text{ or } P(\text{Ace}) = P(\text{King}) + P(\text{Ace}) = \frac{4}{52} + \frac{4}{52} = \frac{8}{52} = \frac{2}{13} \approx .15$$

15%

- Overlapping – events that have at least one common outcome.

(Example)



Common Outcomes

Let's consider a deck of cards.
52 total cards, 4 suits, 13 cards in each suit.

What is the probability that Mya gets a heart or gets a Ace?

→ Overlapping Events = Ace that's heart make sure NOT to count it twice!

$$P(\text{heart}) \text{ or } P(\text{Ace}) = P(\text{heart}) + P(\text{Ace}) = \frac{13}{52} + \frac{(4-1)}{(52-1)} = \frac{13}{52} + \frac{3}{51}$$

$$= \frac{21}{68} \approx .31$$

31%