5.2 Factoring Quadratics, Part 1 $a=1$

GCF Factoring, Difference of Squares & Factoring Trinomials

Standards:

A.SSE.2

A.SSE.3

$\boxed{\text{Old}}$ Multiplying Polynomials

Distribution Property or First Outer Inner Last

Expand the product.

① $(x+3)(x-4)$
$= x^2 - 4x + 3x - 12$
$= x^2 - x - 12$

② $(x-5)(x+5)$
$= x^2 + 5x - 5x - 25$
$= x^2 - 25$

③ $x(7x+6)$
$= 7x^2 + 6x$

④ $4x(3+7x^2)$
$= 12x + 28x^3$

$\boxed{\text{now}}$ Factoring Part 1

Case 1 <u>GCF Factoring</u> – finding the greatest common factor (GCF) of a sum of terms. Use the GCF to create a product.

Let's consider the expression $2x^2(7x+3)$. Expand the expression.

Before FORWARDS (Expand) $\Longleftrightarrow$ Now BACKWARDS (Factor)

$2x^2(7x+3)$
$= 14x^3 + 6x^2$

$14x^3 + 6x^2$
$2x^2(7x+3)$

GCF for #'s

14	6
$1 \cdot 14$	$1 \cdot 6$
$2 \cdot 7$	$2 \cdot 3$

GCF for variable

x^3	x^2
$x \cdot x \cdot x$	$x \cdot x$

[More Examples] Factor the problems.

① $5x^2 + 15x$

$5x(1x+3)$

GCF for #'s

15	5
$1\cdot 15$	$1\cdot 5$
$3\cdot 5$	

GCF for variable

x^2	x
$x\cdot x$	x

② $2x^3 - 8x^2$

$2x^2(x-8)$

GCF for #'s

2	8
$1\cdot 2$	$1\cdot 8$
	$2\cdot 4$

GCF for variable

x^3	x^2
$x\cdot x\cdot x$	$x\cdot x$

③ $2x^2 - 4x$

$2x(x-2)$

GCF for #'s

2	4
$1\cdot 2$	$1\cdot 4$
	$2\cdot 2$

GCF for variable

x^2	x
$x\cdot x$	x

④ $15x^2 - 5x + 30$

$5(3x^2 - x - 6)$

GCF for #'s

15	5	30
$1\cdot 15$	$1\cdot 5$	$1\cdot 30$
$3\cdot 5$		$2\cdot 15$
		$3\cdot 10$
		$5\cdot 6$

GCF for variable

x^2	x	1
$x\cdot x$	x	1

Case B **Factoring Trinomials** – factor the sum of 3 terms (trinomial) into a product of 2 binomials.

Let's consider the expression $(x+3)(x+2)$. Expand the expression.

$$(x+3)(x+2) = x^2 + 2x + 3x + 6$$
$$= x^2 + 5x + 6$$

Standard Form for Trinomials $\Rightarrow$ ax^2+bx+c

$$a x^2 + \underbrace{bx}_{\text{sum}} + \underbrace{c}_{\text{product}}$$

$\boxed{\text{Guess \& Check Method}}$

To factor the trinomial: Find 2 numbers that multiplies to get the "product" (c) & adds up to get the "sum" (b).

Before FORWARDS (Expand) $\Longleftrightarrow$ Now BACKWARDS (Factor)

$(x+3)(x+2)$

$= x^2+2x+3x+6$

$= x^2+5x+6$

Factors of "c"

x^2+5x+6 — $1\cdot 6$, $2\cdot 3$

multiplies to 6 & adds to 5.

$=(x+2)(x+3)$

[Examples] Factor the trinomial.

① $x^2+9x+14$ — $1\cdot 14$, $2\cdot 7$

$=(x+2)(x+7)$

② $n^2-11n+10$ — $-1\cdot -10$, $2\cdot 5$

$=(n-1)(n-10)$

③ $n^2+4n-12$ — $1\cdot 12$, $-2\cdot 6$, $4\cdot 3$

$=(n-2)(n+6)$

④ $m^2+2m-24$ — $1\cdot 24$, $2\cdot 12$, $3\cdot 8$, $-4\cdot 6$

$=(m-4)(m+6)$

⑤ $k^2-13k+40$ — $1\cdot 40$, $2\cdot 20$, $4\cdot 10$, $-5\cdot -8$

$=(k-5)(k-8)$

⑥ $4v^2-4v-8$

$=4(v^2-v-2)$ — $1\cdot 2$

$=4(v-2)(v+1)$

Case C Difference of Squares

Let's consider the expression $(x-5)(x+5)$. Expand the expression.

$(x+5)(x-5) = x^2 - 5x + 5x - 25$
$= x^2 - 25$ ← the middle term (linear term) was eliminated because "5x" and "-5x" are additive inverses.

Difference of Squares mean:
"perfect square" minus "perfect square" $a^2 - b^2 = (\sqrt{a^2} - \sqrt{b^2})(\sqrt{a^2} + \sqrt{b^2})$

Before FORWARD (Expand) ⟷ Now BACKWARD (Factor)

$(x+5)(x-5)$
$= x^2 - 5x + 5x - 25$
$= x^2 - 25$

$x^2 - 25$
$= x^2 + 0x - 25$ $\quad 1 \cdot 25$, $-5 \cdot 5$
$= (x-5)(x+5)$

[Examples]

① $x^2 - 36$
$(x+6)(x-6)$

② $x^2 - 4$
$(x+2)(x-2)$

③ $4w^2 - 16$
$4(w^2 - 4)$
$4(w+2)(w-2)$