5.4 Volumes of Solids of Revolution
Shell Method

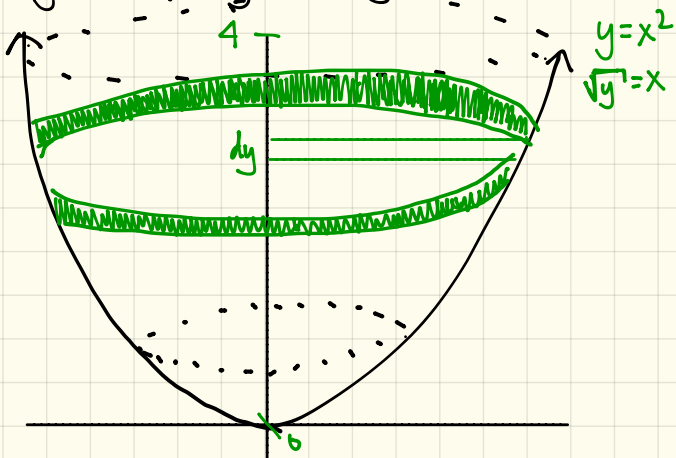
Standards:

MC11

MC11c

Old Volumes of Solids of Revolutions (DISK METHODS)

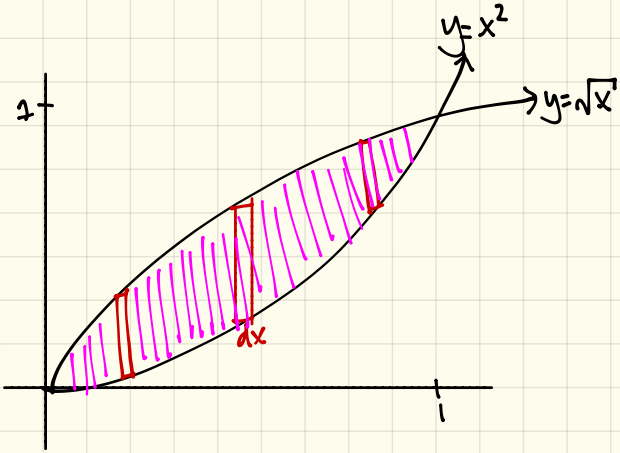
Find the volume of the solid of $y = x^2$ and $x = 0$ and rotate it about the y-axis from $y = 0$ to $y = 4$.



$$V = \int_0^4 \pi (f(y))^2 \, dy = \int_0^4 \pi (\sqrt{y})^2 \, dy = \pi \int_0^4 y \, dy = \pi \left[\frac{y^2}{2}\right]_0^4 = \pi \left[\frac{(4)^2}{2}\right] - \left[\frac{(0)^2}{2}\right]$$

$$= \pi \left[\frac{16}{2}\right] = 4\pi$$

more old... Area between curves



$x^2 = \sqrt{x}$

$(x^2)^2 = (\sqrt{x})^2$

$x^4 = x$

$x^4 - x = 0$

$x(x^3 - 1) = 0$

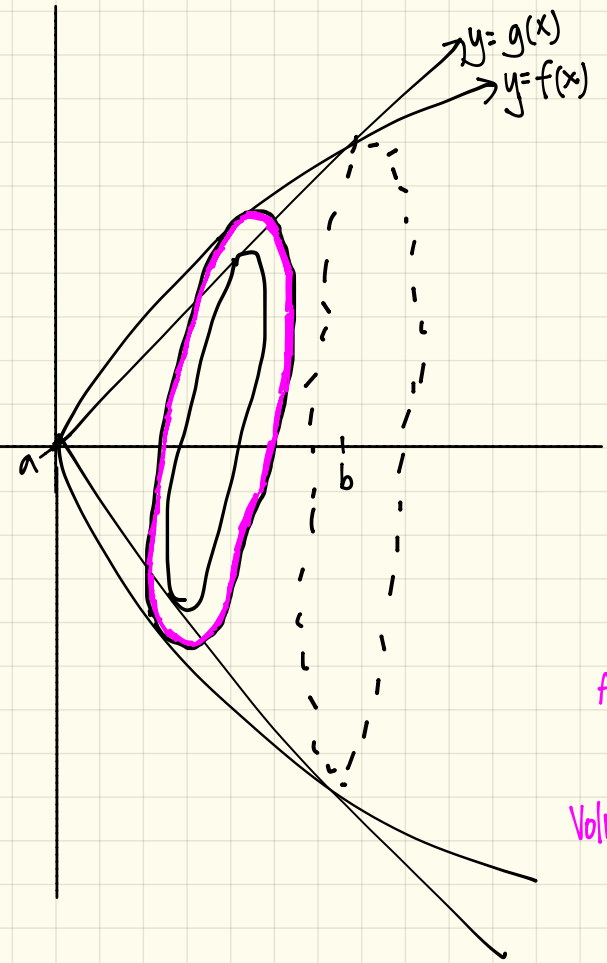
$x = 0, 1$

$$\text{Area} = \int_0^1 [\sqrt{x}] - [x^2]\,dx = \int_0^1 x^{1/2} - x^2\,dx = \frac{x^{3/2}}{3/2} - \frac{x^3}{3}$$

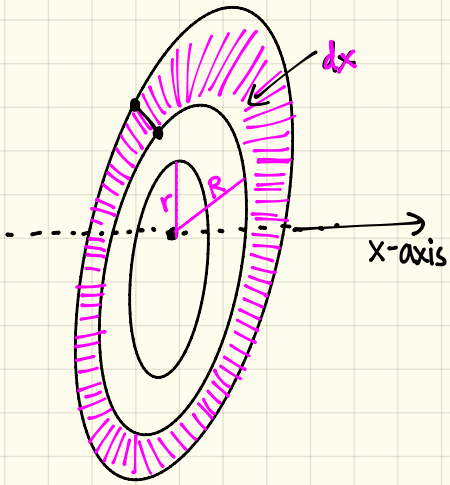
$$= \frac{2}{3}x^{3/2} - \frac{x^3}{3}\Big]_0^1 = \left[\frac{2}{3}(1)^{3/2} - \frac{(1)^3}{3}\right] - \left[\frac{2}{3}(0)^{3/2} - \frac{(0)^3}{3}\right]$$

$$= \frac{2}{3} - \frac{1}{3} = \boxed{\frac{1}{3}}$$

Let's consider the functions $f(x)$ and $g(x)$. Find the volume of revolution of the solid revolving around the x-axis.



A section:



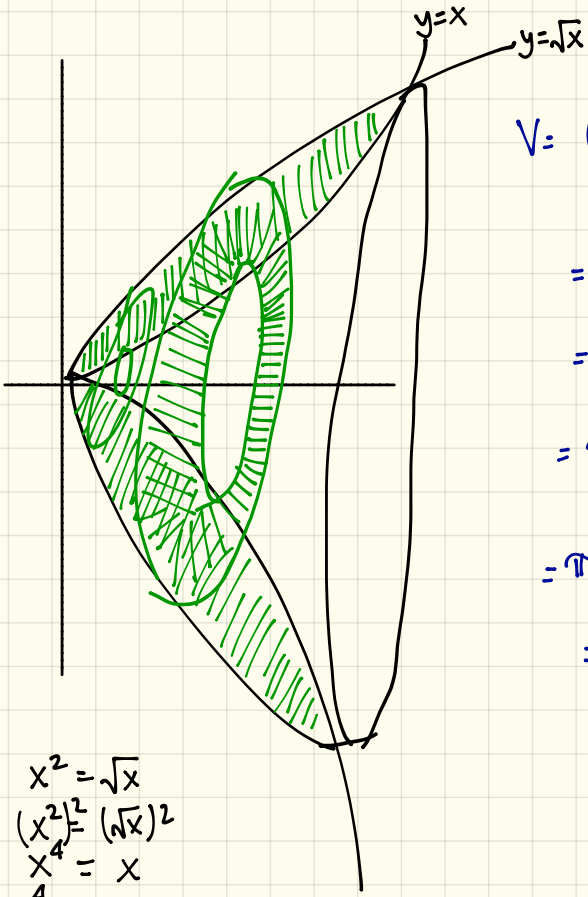
Area $= \pi(R)^2 - \pi(r)^2$
$= \pi(R^2 - r^2)$

Volume = Area * dx
$\pi[R^2 - r^2]\,dx$
} Volume for one section

$$\int_a^b \pi[R^2 - r^2]\,dx$$

Formula for when bounded by 2 curves (WASHER METHOD)

[Example 1] Find the volume of the solid generated about the x-axis by $y=\sqrt{x}$ and $y=x$.



$$V=\int_0^1 \pi\left[(\sqrt{x})^2-(x)^2\right]dx$$

$$=\pi\int_0^1 x-x^2\,dx$$

$$=\pi\left[\frac{x^2}{2}-\frac{x^3}{3}\right]_0^1$$

$$=\pi\left[\frac{(1)^2}{2}-\frac{(1)^3}{3}\right]-\left[\frac{(0)^2}{2}-\frac{(0)^3}{3}\right]$$

$$=\pi\left[\frac{1}{2}-\frac{1}{3}\right]$$

$$=\frac{\pi}{6}$$

$x^2=\sqrt{x}$

$(x^2)^2=(\sqrt{x})^2$

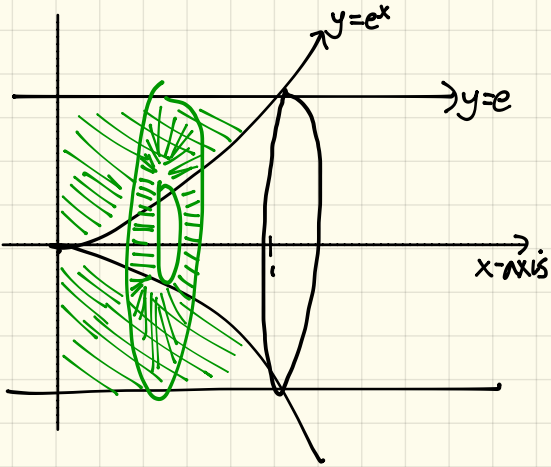
$x^4=x$

$x^4-x=0$

$x(x^3-1)=0$

$x=0,1.$

[Example 2] Find the volume of revolution bounded by $y=e^x$, $y=e$, $x=0$, $x=1$ & revolve about x-axis.



$$V=\int_0^1$$