Definition of Derivative - Classwork

Let us do a bit of reviewing. For the function $f(x) = x^2 - 3x + 1$, find

a) the slope of the tangent line at $x = 2$

b) the slope of the tangent line at $x = 0$.

c) the slope of the tangent line at $x = -1$.

Obviously we have duplicated our efforts a great deal. The process is the same - only the point at which we find the slope of the tangent changes. With that in mind, we are ready to introduce a basic concept of calculus.

The **derivative** of a function is a formula for the slope of the tangent line to that function at any point x. The process of taking derivatives is called **differentiation**.

We now define the derivative of a function $f(x)$ as $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$. This mimics the procedure used above but it calculates the slope of the tangent line at any generic point x. Notice that we can now talk about this process in terms of a limit. When you do your cancellation, you are essentially performing the limit procedures we did in the last section.

Example 1) For the function $f(x) = x^2 - 3x + 1$, find its derivative and evaluate the derivative at $x = 2, 0$, and -1.

We now need to specify some notation for the derivative. When the function is defined as $f(x)$, the derivative will be written as $f'(x)$ or f'. When the function is written in the form of **y =**, the derivative is written as y' or $\frac{dy}{dx}$. The latter looks like a fraction but for now will be one entity, the derivative of y and is pronounced "*dy dx*."

Example 2) $f(x) = 4x$, find $f'(x)$

Example 3) $f(x) = x^2 + x$, find $f'(x)$

Example 4) $f(x) = 2x^2 - 5x + 6$, find $f'(x)$ and $f'(3)$

Example 5) $y = \frac{4}{x}$, find $\frac{dy}{dx}$

For the following functions, find their derivative and evaluate at $x = 2$, -4, 0 and π. Use proper notation.

1. $y = 2x$

Answers: $y' = 2$

$y'(2) = 2 \quad y'(-4) = 2 \quad y'(0) = 2 \quad y'(\pi) = 2$

2. $y = x^2 - 5$

Answers: $y' = 2x$

$y'(2) = 4 \quad y'(-4) = -8 \quad y'(0) = 0 \quad y'(\pi) = 2\pi$

3. $f(x) = x^2 + 3x - 4$

Answers: $f'(x) = 2x + 3$

$y'(2) = 7 \quad y'(-4) = -5 \quad y'(0) = 3 \quad y'(\pi) = 2\pi + 3$

4. $f(x) = 4x^2 - 6x + 1$

Answers: $f'(x) = 8x - 6$

$y'(2) = 10 \quad y'(-4) = -38 \quad y'(0) = -6 \quad y'(\pi) = 8\pi - 6$

5. $f(x) = x^3 + 2x$

Answers: $f'(x) = 3x^2 + 2$

$y'(2) = 14 \quad y'(-4) = 50 \quad y'(0) = 2 \quad y'(\pi) = 3\pi^2 + 2$

6. $f(x) = \dfrac{5}{x} + 1$

Answers: $f'(x) = \dfrac{-5}{x^2}$

$y'(2) = \dfrac{-5}{4} \quad y'(-4) = \dfrac{-5}{16} \quad y'(0) = DNE \quad y'(\pi) = \dfrac{-5}{\pi^2}$

7. $f(x) = \dfrac{-1}{x^2}$

Answers: $f'(x) = \dfrac{2}{x^3}$

$y'(2) = \dfrac{1}{4} \quad y'(-4) = \dfrac{-1}{32} \quad y'(0) = DNE \quad y'(\pi) = \dfrac{2}{\pi^3}$

8. $f(x) = \sqrt{x}$

Answers: $f'(x) = \dfrac{1}{2\sqrt{x}}$

$y'(2) = \dfrac{1}{2\sqrt{2}} \quad y'(-4) = DNE \quad y'(0) = DNE \quad y'(\pi) = \dfrac{1}{2\sqrt{\pi}}$