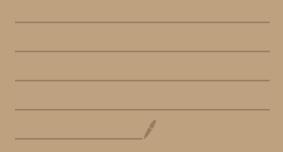
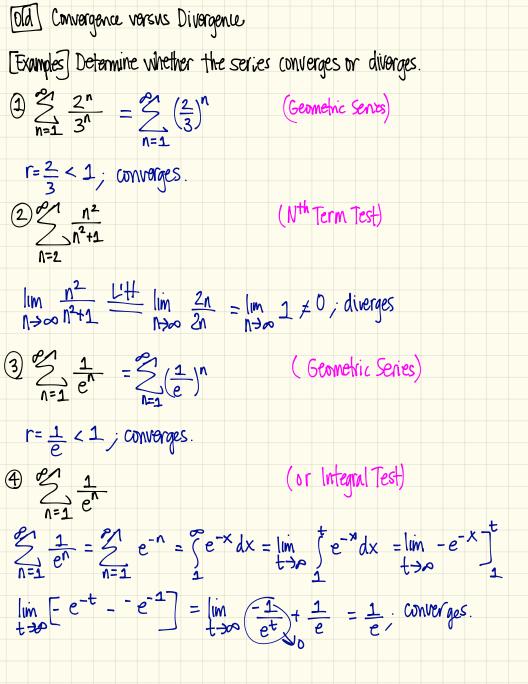
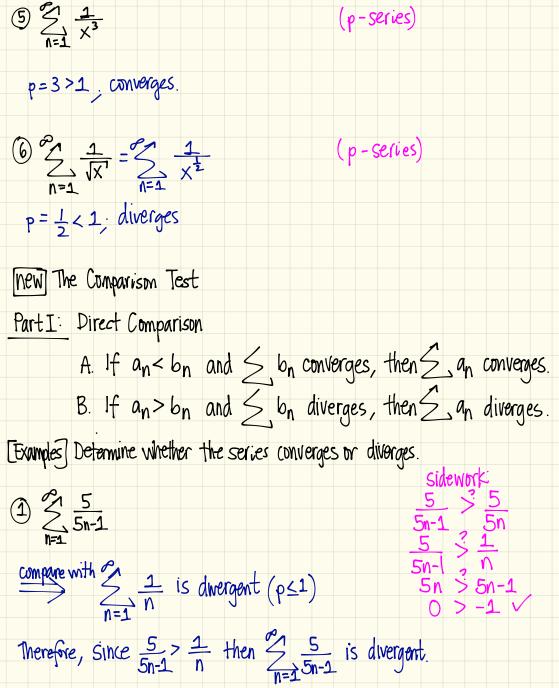
6.4 The Comparison Test







(2) $\sum_{n=1}^{2} \frac{1}{n^3 t_n + 4}$ Sidework: $\frac{1}{n^3 + n + 4} < \frac{1}{n^3}$ compare to p_{1} is convergent (p>1; p-series) $n^{3} < n^{3} + n + 4$ 0 < n + 4Therefore, since $\frac{1}{n^3+n+4} < \frac{1}{n^3}$ and $\leq \frac{1}{n^3}$ is convergent, then 2 1 is convergent. sidework: $3 \sum_{n=1}^{3} \frac{1}{2^{n}-1}$ compared to $p_{n} = \frac{1}{2^{n}} = \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^{n}$ is convergent $(r < 1, \frac{1}{2^{n}} - 1 < 2^{n})$ Therefore, Cuire 1 1 1 Therefore, since $\frac{1}{2^{n-1}} < \frac{1}{2^{n}}$ and $\leq \frac{1}{2^{n}}$ is convergent, then $2 \frac{1}{2^n-1}$ is convergent.

 $(4) \sum_{n=1}^{\infty} \frac{2n^2 + 3n}{\sqrt{5 + n^3}}$ $\begin{array}{c|c} \text{sidework:} \\ \underline{2n^2 + 3n} &> \underline{2n^2} \\ \hline \sqrt{5 + n^5} &> \sqrt{n^5} \\ \underline{2n^2 + 3n} &> \underline{2n^2} \\ \hline \sqrt{5 + n^5} &> \frac{n^5}{2} \\ \underline{2n^2 + 3n} &> \frac{2}{n^{\frac{1}{2}}} \\ \hline \sqrt{5 + n^5} &> \frac{n^{\frac{1}{2}}}{n^{\frac{1}{2}}} \\ \underline{2n^2 + 3n} &> \frac{2}{\sqrt{5 + n^5}} \\ \hline \sqrt{5 + n^5} &> \sqrt{n} \end{array}$ $\sum_{n=1}^{\infty} \frac{2}{\sqrt{n^2}} = \sum_{n=1}^{\infty} \frac{2}{n^{\frac{1}{2}}}$ is divergent (p<1; p-series) Since $\frac{2n^2+3n}{\sqrt{5+n^3}} > \frac{2}{\sqrt{n^2}}$ and $\frac{2}{n=1} = \frac{2}{\sqrt{n}}$ is divergent VS+ms then $\underset{n=1}{\overset{2n^2+3n}{,}}$ is divergent.

Part II: Limit Comparison Let Zan and Zbn be series with positive terms. Then, if lim <u>an</u> = C with c>0, then both series converges n=0 bn or both series durerges. [Examples] Determine whether the series converges or diverges. sidework: $(1) \sum_{n=1}^{\infty} \frac{1}{n^2 - 1}$ Try DIRECT COMPARISON: $\frac{1}{n^{2}-1} >^{?} \frac{1}{n^{2}}$ $n^{2} > n^{2}-1$ * Although the inequality statement is true, $2:\frac{1}{n^2}$ converges (but $a_n > b_n$) so can't use direct companison. $\implies a_n = \frac{1}{n^2 - 1} \neq b_n = \frac{1}{n^2}$ $\lim_{n \to \infty} \frac{1}{n^2 - 1} = \lim_{n \to \infty} \frac{n^2}{n^2 - 1} = \lim_{n \to \infty} \frac{n^2}{n^2 - 1} = \lim_{n \to \infty} \frac{1}{n^2 - 1} = \lim_{n \to \infty} \frac{1}{n^2 - 1} = \lim_{n \to \infty} \frac{1}{n + \infty} = \lim_{n \to \infty} \frac{1}{n + \infty} = 1 > 0;$ limit-companism can be used. 2 1 Converges (p>1; p-series) Since $\lim_{n \to \infty} \frac{a_n}{b_n} > 0$, and $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges, then by limit comparison $\sum_{n = 1}^{2} \frac{1}{n^{2} - 1}$ (onvergences.) n = 10 was created by Keenan Xavier Lee, 2013. See my website for more information, lee-apcalculus.weebly.com.

(2) $\binom{n^3+1}{n^4-5n-5}$ Let $a_n = \frac{n^3 + 1}{n^4 - 5n - 5}$ and $b_n = \frac{n^3}{n^4} = \frac{1}{n}$ $\xrightarrow{n^{3}+1} \lim_{n^{4}-5n-5} \lim_{n \to \infty} \frac{n(n^{3}+1)}{n^{4}-5n-5} = \lim_{n \to \infty} \frac{n(n^{3}+1)}{n^{4}-5n-5} = \lim_{n \to \infty} \frac{n^{4}+n}{n^{4}-5n-5}$ $\frac{L'H}{h \to \infty} \lim_{n \to \infty} \frac{4n^3 + 1}{4n^3 - 5} \frac{L'H}{h \to \infty} \lim_{n \to \infty} \frac{12n^2}{12n^2} = 1 > 0$ Limit comparison can be used 2 1 diverges (p<1; p-series) Since $\lim_{n \to \infty} \frac{a_n}{b_n} > 0$ and $\frac{21}{n} \frac{1}{n}$ diverges, then by limit companison $\sum_{n=1}^{\infty} \frac{n^3 + 1}{n^7 - 5n - 5} \text{ diverges.}$ $(4) \sum_{h=1}^{2} \frac{2}{h^{3}-4}$ Let $a_{n} = \frac{2}{n^{3}-4}$ and $b_{n} = \frac{1}{n^{3}}$ $= \lim_{n \to \infty} \frac{2}{n^3 - 4} = \lim_{n \to \infty} \frac{2n^3}{n^3 - 4} = 2 > 0. \text{ limit comparison can be used}$

^eZ <u>1</u> converges (p>3; p-series) n=1 n³

Since $\lim_{n \to \infty} \frac{a_n}{b_n} > 0$ and $\sum_{n=1}^{\infty} \frac{1}{n^3}$ converges, then by limit companism

 $\frac{2}{2}\frac{2}{n^3-4}$ converges.

SUMMARY for Convergence or Divergence

Divergence Convergence 1. Geometric Serves: if |r| > 1 for $a_p = a(r)^n$, then $\leq a_n$ diverges. 1. Geometric Series: if |r| < 1 for $a_{h} = a(r)^{n}$, then $\leq a_{n}$ converges at a_{1-r} 2. Nth Term Test 2. Nth Term Test: if $\lim_{n \to \infty} a_n \neq 0$, then $\leq a_n$ diverges. if p>1, 2 1 converges 3. p-series: if p≤1, ≤ 1 diverges 4. The Integral Test: 4. The Integral Test: H (fox) dx diverges, then E, a, diverges. if Stadx converges, then Zak converges 5. The Comparison Test 5 The Comparison Test (Direct Comparison) (Direct Comparison) if $a_n > b_n$ and S, b_n diverges, then S, a_n diverges. if an < bn and Z, bn converges, then Z, an converges. (Limit Comparison) if $\lim_{n \to \infty} \frac{a_n}{b_n} = c > 0$ and \leq , b_n converges if $\lim_{n \to \infty} \frac{a_n}{b_n} = c > 0$ and \leq , b_n diverges then S an diverges. then S, an converges.

Homework page 522] 1-5. 5-8, 10, 15 (Quick Review) (Section Review)