6.7 Rational Root Theorem

$\boxed{\text{Old-A}}$ Solving by Factoring

Ways to Solve by factoring

2 Terms	3 Terms	4 Terms
1. GCF 2. Difference of Squares 3. Cubic Formula Factoring	1. GCF 2. Factoring Trinomials	1. GCF Factoring by Grouping

① $x^2-81=0$

$(x+9)(x-9)=0$

$x=9,-9$

② $x^3+5x^2-4x-20=0$

$x^2(x+5)-4(x+5)=0$

$(x+5)(x^2-4)=0$

$(x+5)(x-2)(x+2)=0$

$x=5,-2,2.$

③ $3x^2-3=0$

$3(x^2-1)=0$

$3(x+1)(x-1)=0$

$x=1,-1.$

④ $x^2+6x+8=0$

$(x+4)(x+2)=0$

$x=-4,-2.$

⑤ $x^3-2x^2-9x+18=0$

$x^2(x-2)-9(x-2)=0$

$(x^2-9)(x-2)=0$

$(x+3)(x-3)(x-2)=0$

$x=3,-3,2.$

$\boxed{\text{new-B}}$ Solve if non-factorable

Ways to solve

Quadratic Formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

2 Terms	3 Terms	4 Terms
1. GCF 2. Difference of Squares 3. Cubic Formula Factoring 4. Take the Square Root	1. GCF 2. Factoring Trinomials 3. Complete the Square 4. Quadratic Formula	1. GCF Factoring by Grouping

① $2x^3 - 8x^2 + 3x - 12 = 0$

$$2x^2(x-4) + 3(x-4) = 0$$

$$(x-4)(2x^2+3) = 0$$

$x - 4 = 0$ or $2x^2 + 3 = 0$

$x = 4$ $\quad$ $2x^2 = -3$

$$x^2 = -\frac{3}{2}$$

$$x = \pm i\sqrt{\frac{3}{2}}$$

② $x^4 - 9 = 0$

$$(x^2-9)(x^2+9) = 0$$

$$(x+3)(x-3)(x^2+9) = 0$$

$x + 3 = 0$ or $x - 3 = 0$ or $x^2 + 9 = 0$

$x = -3$ $\quad$ $x = 3$ $\quad$ $x^2 = -9$

$$x = \pm i\sqrt{3}$$

③ $x^3 - 8 = 0$

$$x^3 - 2^3 = 0$$

$$(x-2)(x^2+2x+4) = 0$$

$x - 2 = 0$ or $x^2 + 2x + 4 = 0$

$x = 2$ $\quad$ $x = \dfrac{-2 \pm \sqrt{2^2 - 4(1)(4)}}{2(1)}$

$$= \frac{-2 \pm \sqrt{-12}}{2}$$

$$= \frac{-2 \pm 2i\sqrt{3}}{2}$$

$$= -1 + i\sqrt{3} \text{ or } -1 - i\sqrt{3}.$$

$\boxed{\text{old-B}}$ Long Division

① $x+3 \overline{) x^2+7x+12}$, quotient $x+4$

$\ominus x^2 \ominus 3x$

$4x+12$

$\ominus 4x \ominus 12$

$0.$

② $\dfrac{x^4+4x^2-45}{x^2+9} = x^2+0x+9 \overline{) x^4+0x^3+4x^2+0x-45}$, quotient x^2-5

$\ominus x^4 \ominus 0x^3 \ominus 9x^2$

$-5x^2+0x-45$

$\oplus 5x^2 \oplus 0x \oplus 45$

$0.$

③ $\dfrac{x^2-9}{x-3} = x-3 \overline{) x^2-0x-9}$, quotient $x+3$

$\ominus x^2 \oplus 3x$

$-3x-9$

$\oplus 3x \oplus 9$

$0.$

④ $\dfrac{x^3-4x}{x+2} = x+2 \overline{) x^3+0x^2-4x+0}$, quotient x^2-2x

$\ominus x^2 \ominus 2x^2$

$-2x^2-4x$

$\oplus 2x^2 \oplus 4x$

$0.$

$\boxed{\text{new-B}}$ Rational Root Thm.

Let's consider the function below. Solve for the roots.

$$2x^3-3x^2-11x+6$$

Dilemma - We don't have a factoring technique to use
- We can't divide the polynomial to get the other factors because we don't have anything to divide with.

How can we solve polynomials when given no factors?

$\boxed{\text{Rational Root Thm.}}$ $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x^1 + a_0$ when $a_n \neq 0$.

a_n: Leading coefficient $\qquad a_0$: constant

$\pm\left(\dfrac{\text{factors of constant}}{\text{factors of leading coefficient}}\right) \Leftarrow$ This will give you all possible zeros where you determine the zeros.

[Example] Find the zeros.

$f(x) = \underline{2}x^3 - 3x^2 - 11x + \underline{6}$

6: 1, 2, 3, 6
2: 1, 2

Possible Rational Roots: $\pm\left\{\frac{1}{1}, \frac{1}{2}, \frac{2}{1}, \frac{2}{2}, \frac{3}{1}, \frac{3}{2}, \frac{6}{1}, \frac{6}{2}\right\}$

$\pm\left\{1, \frac{1}{2}, 2, 3, \frac{3}{2}, 6\right\}$

What does it mean to be a root of a polynomial?
When substituting roots into polynomial, the y-value will equal zero (x-intercept).

$x=3$, $x=\frac{1}{2}$ and $x=-2$ are roots. (use calculator to calculate)

[Example 2] $x^3 - 8$

8: 1, 8, 4, 2
1: 1

$\pm\{1, 8, 4, 2\}$