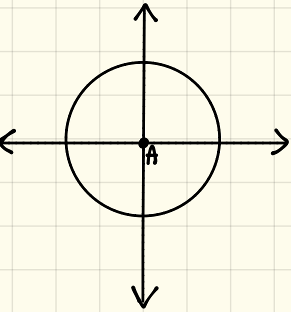
7.1 Introduction to Parametric Equations

old-A Objects on 2d plane

Let's recall a circle on a coordinate plane:



Standard Form of a Circle
$$(x-h)^2+(y-k)^2=r^2$$

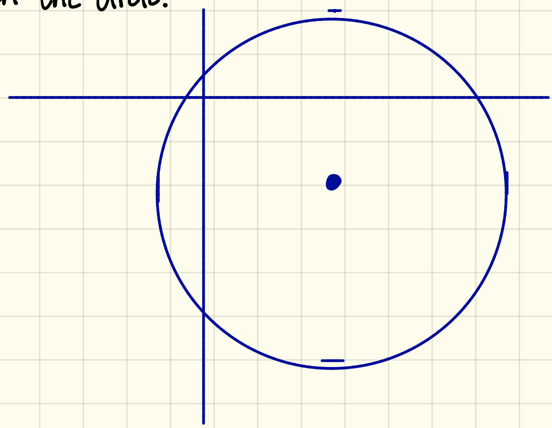
Center of a circle: (h,k)
Radius of a circle: r

[Example 1] Write an equation of a circle with center $(3,-2)$ and radius of 4. Sketch the circle.

Standard Form of a Circle
$$(x-h)^2+(y-h)^2=r^2$$

Center: $(3,-2)$, radius $=4$

$$= (x-3)^2+(y-(-2))^2=(4)^2$$
$$= (x-3)^2+(y+2)=(4)^2$$
$$= (x-3)^2+(y+2)^2=16$$



Let's recall a parabola on a coordinate plane:
A parabola opens up in 4 ways?

a is pos.

a is neg.

(opens up or down) $y=a(x-h)^2+k$
vertex: (h,k)

a is pos.

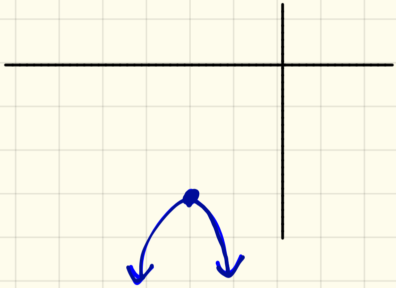
a is neg.

(opens right or left) $x=a(y-k)^2+h$
vertex: (h,k)

[Examples] Sketch the graph.

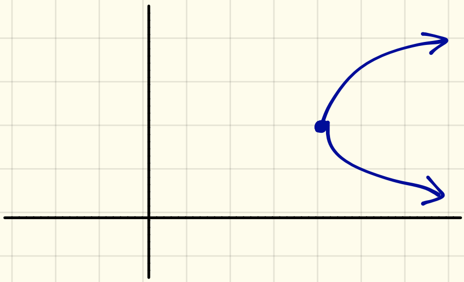
① $y = -(x+2)^2 - 3$

vertex: $(-2,-3)$
no stretch or shrink
(reflection across x-axis



② $x = (y-2)^2 + 4$

vertex: $(4,2)$
no stretch or shrink



$\boxed{\text{old-B}}$ Systems of Equation

Let's recall the techniques for solving systems of equations:
$\boxed{1}$ Graphing $\boxed{2}$ Substitution $\boxed{3}$ Elimination

[Example] $\begin{cases} -2x + y = 1 \\ y = -2x - 3 \end{cases}$. Solve the system.

$$\begin{cases} -2x + y = 1 \\ y = -2x - 3 \end{cases}$$

$$\Longrightarrow -2x + (-2x-3) = 1$$
$$-2x - 2x - 3 = 1$$
$$-4x - 3 = 1$$
$$-4x - \cancel{3} + \cancel{3} = 1 + 3$$
$$\frac{-\cancel{4}x}{-\cancel{4}} = \frac{4}{-4}$$
$$x = -1$$

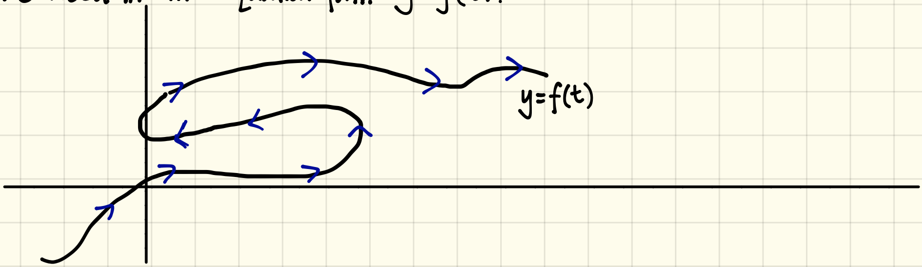
Sub -1 for x:

$$y = -2x - 3$$
$$y = -2(-1) - 3$$
$$y = 2 - 3$$
$$y = -1.$$

Solution: $(-1,-1)$.

$\boxed{\text{new}}$ Intro to Parametric Equations

Let's consider to a particle moving along a curve. Let's say it's a curve described in the equation form $y=f(t)$.



What are some issues that arise with this idea?

- fails vertical line test
- modeling the curve through just the variables of x & y seem impossible

Dilemma: How do we model such curves algebraically?

In order to model this curve algebraic, we must consider a third variable time.

__Definition:__ Parametric Equations

Suppose that are both given as functions of a third variable t (called parameter) by the equations

$$x=f(t) \text{ and } y=g(t) \text{ — parametric equations.}$$

- each value of t determines a point (x,y) which we can plot in a coordinate plane
- as t varies, so will $(x,y) = (f(t), g(t))$ tracing a curve (parametric curve)
- t usually denotes time in which the object (particle) is moving along the curve.

$\boxed{\text{note:}}$ Sometimes we restrict t to lie in a finite interval.

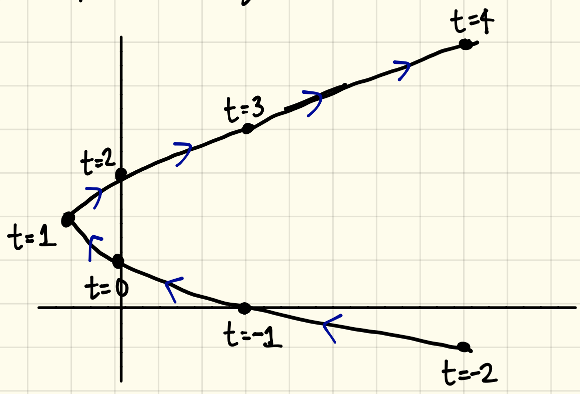
(i.e.) $x=f(t)$ $y=g(t)$ $a \le t \le b$.

initial point $(f(a), g(a))$ — start point

terminal point $(f(b), g(b))$ — end point

Now, let's consider the parametric equations $x = t^2+2t$ and $y=t+1$ between $x=-2$ and 4. Sketch & identify the curve defined by the parametric equations.

t	$x=t^2-2t$	$y=t+1$
-2	8	-1
-1	3	0
0	0	1
1	-1	2
2	0	3
3	3	4
4	8	5



$x=t^2-2t$ and $y=t+1$

$\Rightarrow y=t+1$
$t=y-1$

$\Rightarrow x=t^2-2t$
$=(y-1)^2-2(y-1)$
$=y^2-2y+1-2y+2$
$=y^2-4y+3$

[Examples] Sketch & identify the parametric equations.

① $x=t$ and $y=t^2$; $0 \le t \le 3$.

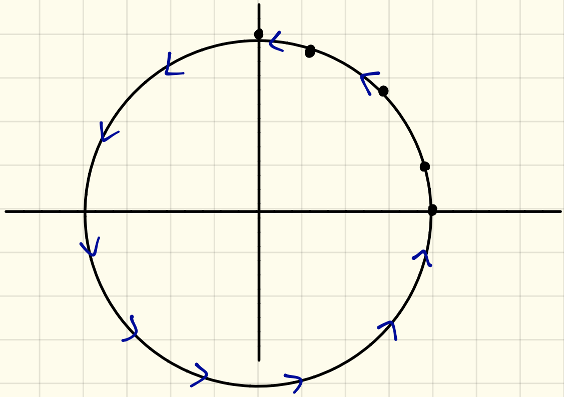
t	x	y
0	0	0
1	1	1
2	2	4
3	3	9

$x=t \quad y=t^2$

$\Rightarrow y=(x)^2$

② $x=\cos t$, $y=\sin t$; $0 \le t \le 2t$.

t	x	y
0	0	1
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
$\frac{\pi}{2}$	1	0
$\vdots$	$\vdots$	$\vdots$



$x=\cos t$ $\quad$ $y=\sin t$ $\longrightarrow$ we know circles are modeled by $x^2+y^2=r^2$

(square both sides)

$(x)^2=(\cos t)^2 \qquad (y)^2=(\sin t)^2$

$x^2=\cos^2 t \qquad y^2=\sin^2 t$

$x^2+y^2=r^2$

$(\cos^2 t)+(\sin^2 t)=(1)^2$

$\Rightarrow \cos^2 t+\sin^2 t=1$.

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