

7.1 Multiplying & Dividing Rationals

Old-A Exponential Rules

Product Rule: $x^a \cdot x^b = x^{a+b}$

Power Raised to a Power: $(x^a)^b = x^{ab}$

Negative Exponent: $x^{-n} = \frac{1}{x^n}$

Quotient Rule: $\frac{x^a}{x^b} = x^{a-b}$

[Examples] Simplify.

(1) $(-3c^4)^2 = 9c^8$

(2) $(4x^2y^3)^3 = 64x^6y^9$

(3) $\frac{x^{12}y^4}{x^{20}y} = x^{-8}y^3 = \frac{y^3}{x^8}$

(4) $\frac{24x^4y^6}{-8x^3y^7} = -3xy^{-1} = -\frac{3x}{y}$

(5) $\frac{1}{9x^2y^{-1}} = \frac{x^2y^1}{9}$

(6) $(4a^2b)^3 \cdot (ab)^3$
 $= 64a^6b^3 \cdot a^3b^3$
 $= 64a^9b^6$

(7) $\left(\frac{3x^2z^4}{2xz}\right)^3 = \frac{27x^6z^{12}}{8x^3z^3} = \frac{27x^3z^9}{8}$

(8) $\frac{3a^7b^{-5}}{27a^{-3}b^8} = \frac{3a^7a^3}{27b^8b^5} = \frac{1a^{10}}{9b^{13}}$

Old-B Factor.

(9) $x^2 - 2x - 8 = (x-4)(x+2)$

(10) $x^2 - 5x = x(x-5)$

(11) $x^5 - 9x^3 = x^3(x-3)(x+3)$

(12) $x^2 - x - 6 = (x-3)(x+2)$

(13) $x^2 - 25 = (x-5)(x+5)$

(13) $x^3 - 8 = (x-2)(x^2 + 2x + 4)$

Old-C Simplifying Fractions

$$\textcircled{1} \frac{9}{24} = \frac{\cancel{3}(3)}{\cancel{8}(3)} = \frac{3}{8}$$

$$\textcircled{2} \frac{4}{16} = \frac{\cancel{(1)}(4)}{\cancel{(4)}(4)} = \frac{1}{4}$$

$$\textcircled{3} \frac{75}{100} = \frac{3(\cancel{25})}{4(\cancel{25})} = \frac{3}{4}$$

New A,B,C Simplifying Rational Expressions

What is a rational expression? A rational expression is a quotient of 2 polynomials.

(examples) $\frac{x^2-4}{x-2}$

$$\frac{10}{x^2-6}$$

$$\frac{x+3}{x-7}$$

[Examples-A] Simplify the rational expression.

$$\textcircled{1} \frac{10x^8}{6x^4} = \frac{5x^4}{3}$$

$$\textcircled{2} \frac{9x^{10}}{24x^{15}} = \frac{3x^{-5}}{8} = \frac{3}{8x^5}$$

$$\textcircled{3} \frac{4x^{15}}{16x^{16}} = \frac{1}{4}x^{-1} = \frac{1}{4x}$$

[Examples-B] Simplify the rational expression.

$$\textcircled{1} \frac{x^2-4}{x-2} = \frac{(x+2)\cancel{(x-2)}}{1\cancel{(x-2)}} = x+2$$

$$\textcircled{2} \frac{x^2+x-12}{x^2+10x+24} = \frac{(x-3)\cancel{(x+4)}}{\cancel{(x+6)}(x+4)} = \frac{x-3}{x+6}$$

$$\textcircled{3} \frac{3x+4}{3x^2+x-4} = \frac{1\cancel{(3x+4)}}{\cancel{(3x+4)}(x-1)} = \frac{1}{(x-1)}$$

$$\textcircled{4} \quad \frac{x^2 - 4x}{x^2 - 2x - 8} = \frac{x(x-4)}{(x-4)(x+2)} = \frac{x}{x+2}$$

$$\textcircled{5} \quad \frac{2x-10}{x-5} = \frac{2(x-5)}{1(x-5)} = 2.$$

Std-D Multiplying Fractions (cross reducing is important!)

$$\textcircled{1} \quad \frac{5}{\cancel{16}} \cdot \frac{\cancel{12}^2}{8} = \frac{5}{1} \cdot \frac{2}{8} = \frac{10}{8} = \frac{5(\cancel{2})}{4(\cancel{2})} = \frac{5}{4}.$$

$$\textcircled{2} \quad \frac{\cancel{12}^2}{\cancel{7}^1} \cdot \frac{\cancel{21}^3}{\cancel{4}^2} = \frac{3}{2}.$$

$$\textcircled{3} \quad \frac{1}{2} \div \frac{3}{4} = \frac{1}{2} \cdot \frac{\cancel{4}^2}{3} = \frac{2}{3}.$$

$$\textcircled{4} \quad \frac{7}{6} \div \frac{14}{8} = \frac{\cancel{7}^1}{\cancel{6}^3} \cdot \frac{\cancel{8}^4}{\cancel{14}^2} = \frac{4}{6} = \frac{2}{3}.$$

$$\textcircled{5} \quad \frac{8}{10} \div \frac{5}{20} = \frac{\cancel{8}^4}{\cancel{10}^5} \cdot \frac{\cancel{20}^2}{5} = \frac{16}{5}.$$

Now - D Multiplying & Dividing Rational Expressions

Steps

1. Factor all numerators & denominators completely.
 2. Divide out common factors of the numerator & divide denominators.
 3. Multiply numerators. Divide denominators.
- (Be sure the numerator & denominator have no other common factor other than 1.)

[Examples] Multiply.

$$\textcircled{1} \frac{x-3}{4x+20} \cdot \frac{x+5}{x^2-9}$$
$$= \frac{1(x-3)}{4(x+5)} \cdot \frac{1(x+5)}{(x+3)(x-3)}$$
$$= \frac{1}{4(x+3)}$$
$$\textcircled{2} \frac{10x-40}{x^2-6x+8} \cdot \frac{x+3}{5x+15}$$
$$= \frac{\cancel{10}(x-4)}{(x-4)(x-2)} \cdot \frac{1(x+3)}{\cancel{5}(x+3)}$$
$$= \frac{2}{x-2}$$

[Examples] Divide.

$$\textcircled{1} \frac{x^4-9x^2}{x^2-4x+3} \div \frac{x^4+2x^3-8x^2}{x^2-16}$$
$$\frac{x^4-9x^2}{x^2-4x+3} \cdot \frac{x^2-16}{x^4+2x^3-8x^2}$$

$$\frac{x^2(x^2-9)}{(x+3)(x-1)} \cdot \frac{(x+4)(x-4)}{x^2(x^2+2x-8)}$$

$$\frac{x^2(x+3)(x-3)}{(x+3)(x-1)} \cdot \frac{(x+4)(x-4)}{x^2(x-2)(x-4)}$$

This was created by Keenan Xavier Lee - 2014. See my website for more information, lee-apcalculus.weebly.com.

$$\frac{(x+3)(x+4)}{(x-1)(x-2)}$$