7.1 Substitution Rule for Integration

Standards:

MC11

MC11d

$\boxed{01d}$ Integration using power rule for antiderivatives

Recall: In order to apply power rule for antiderivatives, the function must be sum.

① $\int x^2 - \sin x\,dx = \frac{x^3}{3} + \cos x + C$

② $\int x^2 + \sqrt{x}\,dx =$ Rewrite $= \int x^2 + x^{1/2}\,dx = \frac{x^3}{3} + \frac{x^{3/2}}{\frac{3}{2}} + C$

$= \frac{1}{3}x^3 + \frac{2}{3}x^{3/2} + C$

$= \frac{1}{3}x^3 + \frac{2\sqrt{x^3}}{3} + C$

③ $\int_0^4 \frac{1-\sqrt{u}}{\sqrt{u}}\,du = \int_0^4 \frac{1-u^{1/2}}{u^{1/2}}\,du = \int_0^4 \frac{1}{u^{1/2}} - \frac{u^{1/2}}{u^{1/2}}\,du$

$= \int_0^4 u^{-1/2} - 1\,du = \frac{u^{1/2}}{1/2} - u = 2u^{1/2} - u \Big]_0^4$

$= \left[2(4)^{1/2} - (4)\right] - [0] \quad = 0.$

$\boxed{\text{new}}$ Substitution Rule for Integration

Now: What if we have to deal with composition of functions?

Recall → Chain Rule

$$\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)$$

Now let's integrate this:

$$\int \frac{d}{dx} f(g(x)) = \int f'(g(x)) \cdot g'(x)$$

$$f(g(x)) = \int f'(g(x)) \cdot g'(x) \blacksquare$$

Basic Idea When integrating a composition of functions, to compute them it is the "chain rule in reverse".

U-substition Technique will help compute the "chain rule in reverse".

[Example] $\int 2x(x^2+1)^9 \, dx$

$$\int 2x (x^2+1)^9 \, dx = \int u^9 \, du = \frac{u^{10}}{10} + C = \frac{(x^2+1)^{10}}{10} + C.$$

$u = x^2+1$

$du = 2x \, dx$

Check solution: $y = \frac{(x^2+1)^{10}}{10} = \frac{1}{10}(x^2+1)^{10}$

$y' = \frac{1}{10}(10)\left[x^2+1\right]^9 \cdot 2x = 2x(x^2+1)^9$ ✓

② $\int xe^{x^2}dx = \frac{1}{2}\int e^u du = \frac{1}{2}e^u + c = \frac{1}{2}e^{x^2} + C.$

$u = x^2$
$du = 2x\,dx \Rightarrow \frac{1}{2}du = x\,dx$

Check solution: $y = \frac{1}{2}e^{x^2} + C$
$y' = \frac{1}{2}e^{x^2} \cdot 2x$
$= xe^{x^2}.$

③ $\int 2x\sqrt{1+x^2}\,du = \int \sqrt{u}\,du = \int u^{1/2}du = \frac{u^{3/2}}{3/2} + C$
$= \frac{2}{3}u^{3/2} + C$
$= \frac{2}{3}(1+x^2)^{3/2} + C$
$= \frac{2\sqrt{1+x^2}}{3} + C$

$u = 1+x^2$
$du = 2x\,dx$

④ $\int x^3\cos(x^4+2)\,dx = \frac{1}{4}\int \cos u\,du = \frac{1}{4}\sin u + C$
$= \frac{1}{4}\sin(x^4+2) + C$

$u = x^4+2$
$du = 4x^3 dx \Rightarrow \frac{1}{4}du = x^3 dx$

⑤ $\int \sqrt{2x+1}\,dx \Rightarrow \frac{1}{2}\int \sqrt{u}\,du = \frac{1}{2}\int u^{1/2}\,du = \frac{1}{2}\frac{u^{3/2}}{3/2} + C$

$u = 2x+1$

$du = 2dx \Rightarrow \frac{1}{2}du = dx$

$= \frac{1}{2}\left(\frac{2}{3}\right) u^{3/2} + C$

$= \frac{1}{3}(2x+1)^{3/2} + C$

$= \frac{1\sqrt{(2x+1)^3}}{3} + C$

⑥ $\int \cos 3x\,dx = \frac{1}{3}\int \cos u\,du = \frac{1}{3}\sin u + C$

$u = 3x$

$du = 3\,dx \Rightarrow \frac{1}{3}du = dx$

$= \frac{1}{3}\sin(3x) + C$

Trigonometric Substitution

With integrating trig functions, the basic idea is to:

1. use trig identities
2. use u-substitution technique