7.2 Solving Rational Equations

Old Operations with Rational Equations

① $\frac{15n-15}{15n^3-15n^2} \cdot \frac{3n^2}{n^2+4n-5}$

$= \frac{15(n-1)}{15n^2(n-1)} \cdot \frac{3n^2}{(n+1)(n-5)}$

$= \frac{3}{(n+1)(n-5)}$

② $\frac{2x^2-12x+18}{12x-36} \div \frac{7x+8}{42x+48}$

$= \frac{2x^2-12x+18}{12x-36} \cdot \frac{42x+48}{7x+8}$

$= \frac{2(x^2-6x+9)}{12(x-3)} \cdot \frac{6(7x+8)}{7x+8}$

$= \frac{2(x-3)(x-3)}{12(x-3)} \cdot \frac{6(7x+8)}{7x+8}$

$= (x-3)$.

③ $\frac{2x^2-16}{x^2-4} - \frac{x+4}{x+2}$

$= \frac{2x^2-16}{(x-2)(x+2)} - \frac{x+4}{(x+2)(x-2)} \cdot (x-2)$

$= \frac{2x^2-16}{(x-2)(x+2)} - \frac{x^2+2x-8}{(x+2)(x-2)}$

$= \frac{2x^2-16-(x^2+2x-8)}{(x-2)(x+2)}$

$= \frac{2x^2-16-x^2-2x+8}{(x-2)(x+2)}$

$= \frac{x^2-2x-8}{(x-2)(x+2)} = \frac{(x-4)(x+2)}{(x-2)(x+2)} = \frac{(x-4)}{(x-2)}$

④ $\frac{4}{x+4} - \frac{3}{x-1}$

$= \frac{4}{(x+4)} \cdot \frac{(x-1)}{(x-1)} - \frac{3}{(x-1)} \cdot \frac{(x+4)}{(x+4)}$

$= \frac{4x-4}{(x+4)(x-1)} - \frac{3x+12}{(x-1)(x+4)}$

$= \frac{4x-4-(3x+12)}{(x+4)(x-1)}$

$= \frac{4x-4-3x-12}{(x+4)(x-1)}$

$= \frac{x-16}{(x+4)(x-1)}$

New Solving Rational Equations

STEPS:
1. Identify undefined values
2a. Fraction ± Fraction = Fraction → Multiply each by LCM: eliminate denom!
2b. Fraction = Fraction → cross multiply.
3. Check your answers.

[Example 1]

$$x + \frac{8}{x} = 6$$

$x \neq 0$.
LCM $= x$

$$x(x) + \frac{8(x)}{x} = 6(x)$$

$$x^2 + 8 = 6x$$
$$x^2 - 6x + 8 = 0$$
$$(x-4)(x-2) = 0$$
$$\boxed{x = 4, 2}$$

[Example 2] $\frac{10}{3} = \frac{4}{x} + 2$

$x \neq 0$
LCM $= 3x$

$$\frac{10(3x)}{3} = \frac{4(3x)}{x} + 2(3x)$$
$$10x = 12 + 6x$$
$$4x = 12$$
$$\boxed{x = 3}$$

[Example 3] $\frac{x}{x+1} = \frac{x}{x-4}$

$x \neq -1, x \neq 4$

$$x(x-4) = x(x+1)$$
$$x^2 - 4x = x^2 + 1x$$
$$-x^2 \quad = -x^2$$
$$-4x = 1x$$
$$-1x = -1x$$
$$-5x = 0$$
$$\boxed{x = 0.}$$

[Example 4] $\frac{1}{x-1} = \frac{x}{x-1} + \frac{x}{6}$

$x \neq 1$
LCM $= 6(x-1)$

$$\frac{1[6(x-1)]}{x-1} = \frac{x[6(x-1)]}{x-1} + \frac{x[6(x-1)]}{6}$$
$$6 = 6x + x(x-1)$$
$$6 = 6x + x^2 - 1x$$
$$6 = x^2 + 5x$$
$$0 = x^2 + 5x - 6$$
$$0 = (x+6)(x-1)$$
$$\boxed{x = -6, 1} \leftarrow \text{can't equal 1!}$$