7.4 Graphing Rational Functions

$\boxed{\text{Old-A}}$ Solving Polynomials

① $16x^2 - 8x = 0$
$8x(2x-1) = 0$
$x = 0, \frac{1}{2}$.

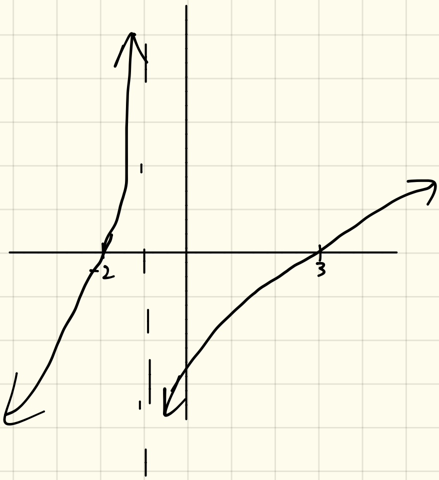
② $x^2 - 4 = 0$
$(x+2)(x-2) = 0$
$x = -2, 2$

③ $x^2 + 5x = 14$
$x^2 + 5x - 14 = 0$
$(x+7)(x-2) = 0$
$x = -7, 2$

note: These solutions represents the x-intercepts (or zeros, roots) of the graphs.

$\boxed{\text{New-A}}$ Graphing Rational

Let's consider the rational function: $f(x) = \dfrac{(x-3)(x+2)}{x+1}$. Let's look at the graph & analyze.



- The vertical asymptote is at $x = -1$.
- The zeros (x-intercepts) are $x = -2$ and $x = 3$.

where do the values come from (algebraically)?
- zeros are from the numerator being set equal to 0.
- vertical asymptote are from the denominator being set equal to 0.

Let's generalize!

Rational Function $\Rightarrow \dfrac{f(x)}{g(x)}$

$f(x) = 0$ will get the x-intercepts.
$g(x) = 0$ will get the vertical asymptote.

[Examples] Identify the zeros & vertical asymptotes.

① $f(x) = \frac{x^2+3x-4}{x+3}$

zeros:
$x^2+3x-4=0$
$(x+4)(x-1)=0$
$x=-4, 1$
$(-4,0)\ (1,0)$

vertical asymptotes:
$x+3=0$
$x=-3.$

② $g(x) = \frac{x^2+7x+6}{x-5}$

zeros:
$x^2+7x+6=0$
$(x+6)(x+1)=0$
$x=-6, -1$
$(-6,0)\ (-1,0)$

vertical asymptotes:
$x-5=0$
$x=5.$

$\boxed{\text{new - B}}$ Horizontal Asymptotes, y-intercept

- To find y-intercepts: substitute 0 in for x and evaluate.
- To find the horizontal asymptotes: expand the polynomials in the numerator & denominator
  - a) If the degree is higher in the numerator → No Horizontal Asym.
  - b) If the degree is higher in the denominator → $y=0$.
  - c) If the degree is the same in the numerator & denominator
    ↳ $y = \frac{\text{leading coefficient}}{\text{leading coefficient}}$

[Examples] Determine the x-intercept, y-intercept, Horizontal Asym & vertical Asym.

① $f(x) = \frac{3x-4}{x-2}$

x-int: $(\frac{4}{3}, 0)$
y-int: $(0, 2)$
HA: $y=3$
VA: $x=2$

② $f(x) = \frac{2}{x^2-4}$

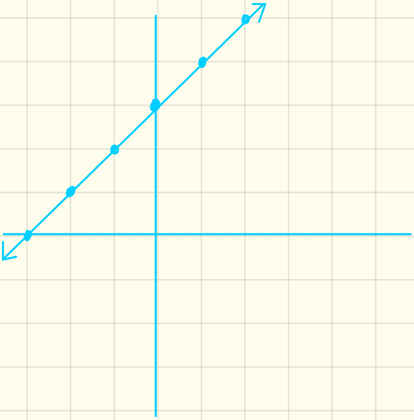
x-int: None
y-int: $(0, -\frac{1}{2})$
HA: $y=0$
VA: $x=2, x=2.$

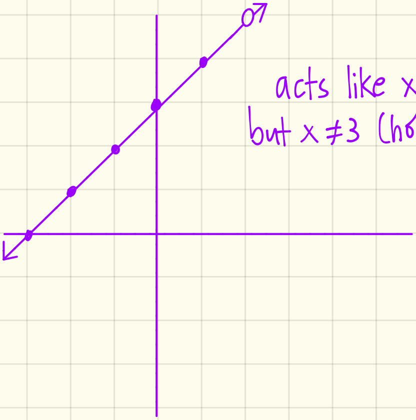
③ $f(x) = \frac{x^2+2x-15}{x^2-4}$

x-int: $(-5,0)\ (3,0)$
y-int: $(0, 3.75)$
HA: $y=1$
VA: $x=-2, x=2$

new - C Holes, Slant Asymptote

Let's consider $f(x) = x+3$ & $g(x) = \frac{x^2-9}{x-3}$. Graph & analyze.





acts like $x+3$
but $x \neq 3$ (hole)

To identify "holes" in function, determine the factors that need to be eliminated in the numerator & denominator. Set that factor equal to 0.

$$g(x) = \frac{x^2-9}{x-3} = \frac{(x+3)\cancel{(x-3)}}{\cancel{x-3}} = x+3$$

$x - 3 = 0$
$x = 3$. ← hole at $x=3$

[Examples] Where are the holes in each?

① $f(x) = \frac{(x-3)\cancel{(x+2)}}{\cancel{(x+2)}(x-1)} = \frac{x-3}{x-1}$

$x+2=0$
$x=-2$. ← hole!

② $f(x) = \frac{2x^2+4x}{x^2+5x+6} = \frac{2x\cancel{(x+2)}}{\cancel{(x+2)}(x+3)}$

$x+2=0$
$x=-2$ ← hole!

Slant Asymptotes occur when the degree of the numerator is exactly one bigger than the degree of the denominator.

$\Longrightarrow$ To find the equation of the slant asymptote, divide the numerator with the denominator [Long Polynomial Division].

[Examples] Find the slant asymptote.

1. $\dfrac{2x^2-3x}{x+1}$

$$
\begin{array}{r l}
 & 2x - 5 + \frac{5}{x+1} \\
x+1 \,\big) & 2x^2 - 3x + 0 \\
 & -2x^2 - 2x \\
\hline
 & -5x + 0 \\
 & +5x + 5 \\
\hline
 & 5
\end{array}
$$

Slant Asym Eqtn: $y = 2x - 5$.