8.1 Area Between Curves

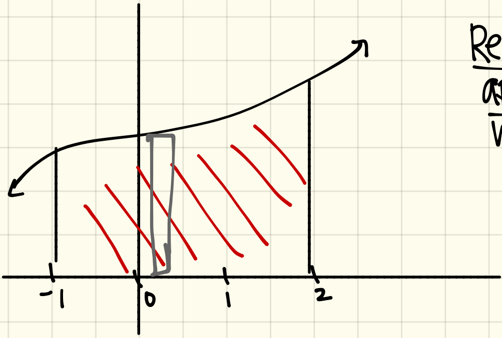
Standards:

MC11

MC11c

$\boxed{\text{Old}}$ Finding Area using Integration

Let's consider the function $f(x)=x^3+2$. Find the area between $x=-1$ & $x=2$.



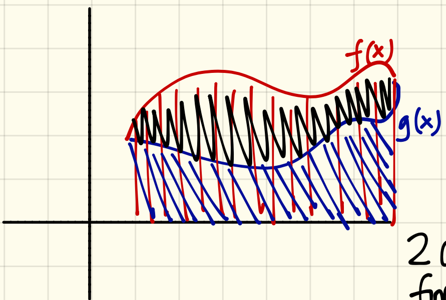
Recall: we can find the area either by approximating with rectangles or computing with Integrals.

$$\int_{-1}^{2} x^3+2\,dx = \frac{x^4}{4}+2x\Big]_{-1}^{2} = \left[\frac{(2)^4}{4}+2(2)\right]-\left[\frac{(-1)^4}{4}+2(-1)\right] = [8]-[-1.75]$$

$$= 9.75.$$

$\boxed{\text{new}}$ Area between curves

Let's consider 2 functions: $f(x)$ & $g(x)$. We want to find the area between 2 arbitrary curves.



$$\int_a^b f(x)\,dx - \int_a^b g(x)\,dx =$$

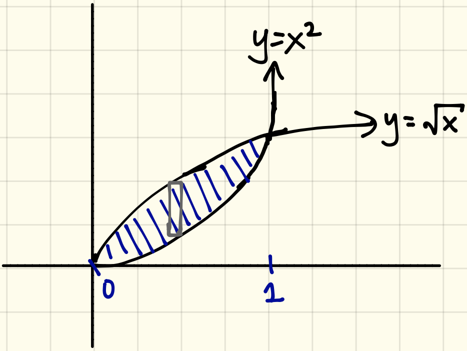
Area under 2 curves formula with respect to x.

$$\boxed{= \int_a^b f(x)-g(x)\,dx}$$

the most top curve

the most bottom curve

[Example 1] Find the area under the curves.



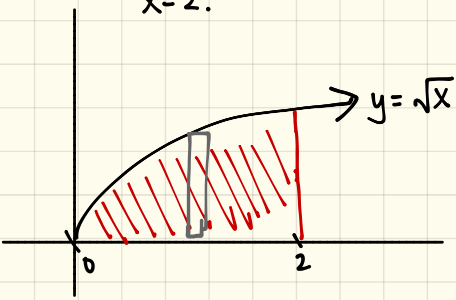
$$\int_0^1 [\sqrt{x}] - [x^2]\,dx = \int_0^1 x^{1/2} - x^2\,dx$$

$$= \frac{2}{3}x^{3/2} - \frac{x^3}{3}\Big]_0^1 = \left[\frac{2}{3}(1)^{3/2} - \frac{(1)^3}{3}\right] - [0]$$

$$= \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$$

$$(\sqrt{x})^2 = (x^2)^2$$
$$x = x^4$$
$$0 = x^4 - x$$
$$0 = x(x^3 - 1)$$
$$x = 0,\ 1$$

[Example 2] Find the area between the curves $y=0$ & $y=\sqrt{x}$ between $x=0$ and $x=2$.

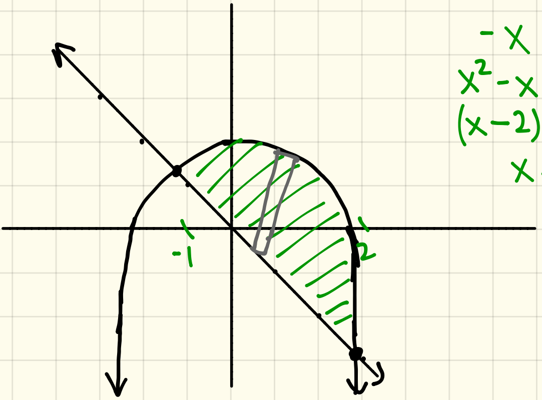


$$\int_0^2 \sqrt{x} - 0\,dx = \int_0^2 x^{1/2}\,dx = \frac{2}{3}x^{3/2}$$

$$= \frac{2}{3}\sqrt{x^3}\Big]_0^2 = \left[\frac{2}{3}\sqrt{(2)^3}\right] - \left[\frac{2}{3}\sqrt{0^3}\right]$$

$$= \frac{2\sqrt{8}}{3}.$$

[Example 3] Find the area between $y = -x$ & $y = 2 - x^2$.



$$-x = 2 - x^2$$
$$x^2 - x - 2 = 0$$
$$(x-2)(x+1) = 0$$
$$x = 2, -1$$

$$\int_{-1}^{2} [2 - x^2] - [-x]\, dx = \int_{-1}^{2} 2 - x^2 + x\, dx = 2x - \frac{x^3}{3} + \frac{x^2}{2}\Big]_{-1}^{2}$$

$$= \left[2(2) - \frac{(2)^3}{3} + \frac{(2)^2}{2}\right] - \left[2(-1) - \frac{(-1)^3}{3} + \frac{(-1)^2}{2}\right]$$

$$= \left[4 - \frac{8}{3} + \frac{4}{2}\right] - \left[-2 + \frac{1}{3} - \frac{1}{2}\right]$$

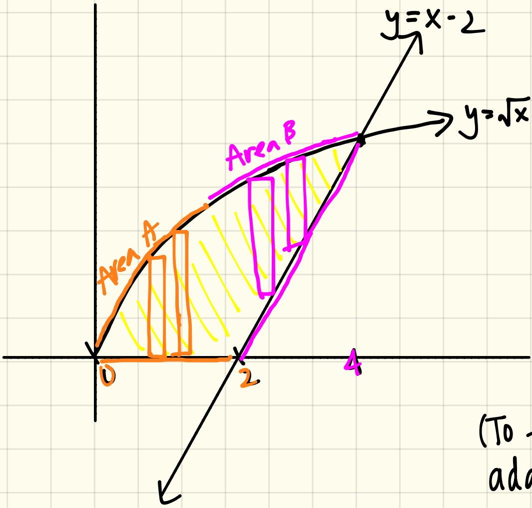
$$= 4 + 2 - \frac{8}{3} - \frac{1}{3} + \frac{4}{2} - \frac{1}{2}$$

$$= 6 - \frac{9}{3} + 2 - \frac{1}{2}$$

$$= 6 - 3 + 2 - \frac{1}{2}$$

$$= 5 - \frac{1}{2} = 4.5$$

[Example 4] Find the area of curves bounded by $y=\sqrt{x}$, $y=0$ and $y=x-2$.



Area A + Area B

$$\text{Area A} = \int_0^2 [\sqrt{x}] - [0]\, dx$$

$$\text{Area B} = \int_2^4 [\sqrt{x}] - [x-2]\, dx$$

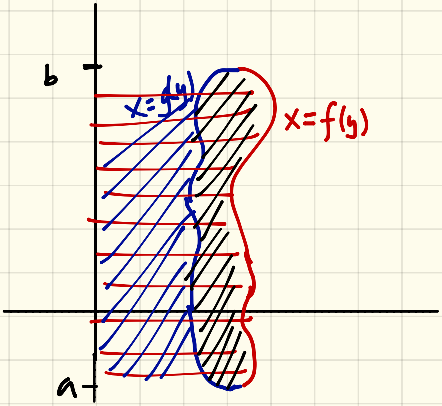
⋯ Do on your own.

(To finish, evaluate both integrals & add the results.)

$x-2=0$
$x=2$

$(\sqrt{x})^2 = (x-2)^2$
$x = (x-2)^2$
$x = x^2-4x+4$
$0 = x^2-5x+4$
$0 = (x-4)(x-1)$
$0 = x=4, \cancel{1}$

Now let's consider the 2 equations: $x = f(y)$ and $x = g(y)$.



$$\int_a^b f(y)\,dy - \int_a^b g(y)\,dy =$$

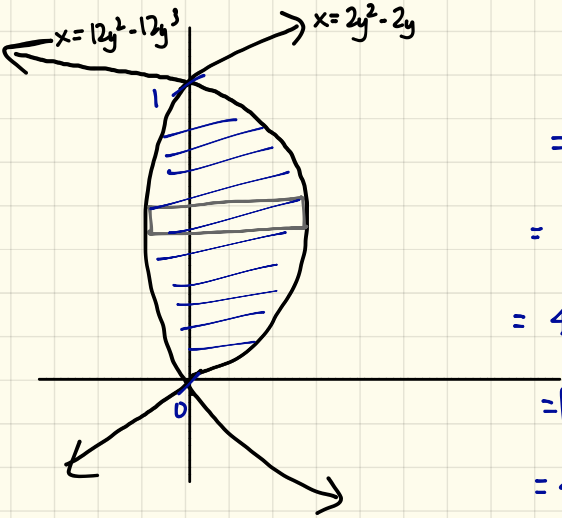
$$\boxed{= \int_a^b f(y) - g(y)\,dy}$$

Area between 2 the 2 curves formula in respect to y

the most right curve

the most left curve

[Example 1] Find the area between curves.



$$\int_0^1 [12y^2 - 12y^3] - [2y^2 - 2y]\,du$$

$$= \int_0^1 12y^2 - 12y^3 - 2y^2 + 2y\,dy$$

$$= \frac{12y^3}{3} - \frac{12y^4}{4} - \frac{2y^3}{3} + \frac{2y^2}{2}$$

$$= 4y^3 - 3y^4 - \frac{2}{3}y^3 + y^2 \Big]_0^1$$

$$= \left[4(1)^3 - 3(1)^4 - \frac{2}{3}(1)^3 + (1)^2\right] - [0]$$

$$= 4 - 3 - \frac{2}{3} + 1$$

$$= 2 - \frac{2}{3} = \frac{6}{3} - \frac{2}{3} = \frac{4}{3}$$