

8.2 Average Value of Function

Standards:

MC11

MC11c



The average value of a function $f(x)$ on the interval $[a, b]$ is:

$$f_{\text{ave}} = \frac{1}{b-a} \int_a^b f(x) dx.$$

[Example 1] Find the average value of $f(x) = 4 - x^2$ on $[0, 3]$.

$$\begin{aligned} f_{\text{ave}} &= \frac{1}{b-a} \int_a^b f(x) dx = \frac{1}{3-0} \int_0^3 4 - x^2 dx = \frac{1}{3} \int_0^3 4 - x^2 dx = \frac{1}{3} \left[4x - \frac{x^3}{3} \right]_0^3 \\ &= \frac{1}{3} \left[4(3) - \frac{(3)^3}{3} \right] - \left[4(0) - \frac{(0)^3}{3} \right] = \frac{1}{3} \left[12 - \frac{27}{3} \right] = \frac{1}{3} [12 - 9] = \frac{1}{3} [6] = 2 \end{aligned}$$

[Example 2] Find the average value of $f(x) = \sin x$ on $[0, \pi]$.

$$\begin{aligned} f_{\text{ave}} &= \frac{1}{b-a} \int_a^b f(x) dx = \frac{1}{\pi-0} \int_0^{\pi} \sin x dx = \frac{1}{\pi} \int_0^{\pi} \sin x dx = \frac{1}{\pi} \left[-\cos x \right]_0^{\pi} \\ &= \frac{-1}{\pi} \left[\cos(\pi) \right] - \left[\cos(0) \right] = \frac{-1}{\pi} \left[-1 \right] - \left[1 \right] = \frac{-1}{\pi} (-2) = \frac{2}{\pi} \end{aligned}$$