8.3 Volumes of Solids of Revolution

Disk Method

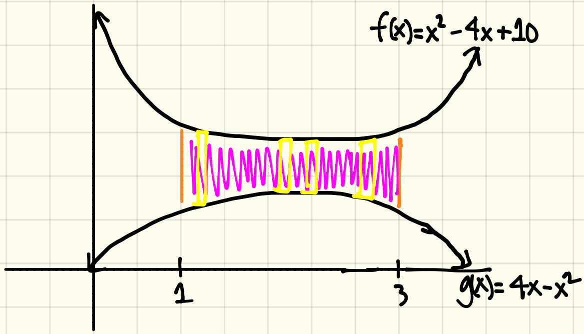
Standards:

MC11

MC11c

$\boxed{\text{Old}}$ Area between curves

Let's consider $f(x) = x^2 - 4x + 10$ & $g(x) = 4x - x^2$. Find the area of region between $x=1$ & $x=3$.



$$\int_1^3 [x^2 - 4x + 10] - [4x - x^2]\,dx$$

$$= \int_1^3 x^2 - 4x + 10 - 4x + x^2\,dx$$

$$= \int_1^3 2x^2 - 8x + 10\,dx$$

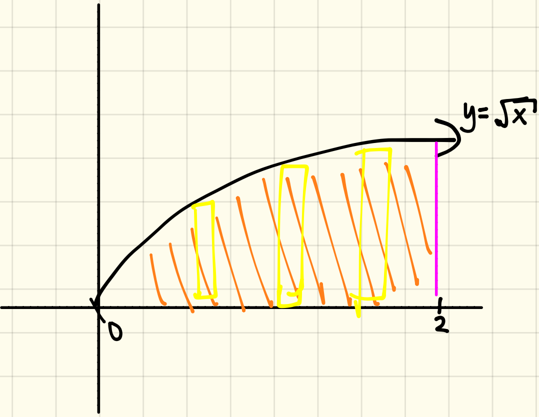
$$= \frac{2x^3}{3} - \frac{8x^2}{2} + 10x = \frac{2}{3}x^3 - 4x^2 + 10x \Big]_1^3 =$$

$$= \left[\tfrac{2}{3}(3)^3 - 4(3)^2 + 10(3)\right] - \left[\tfrac{2}{3}(1)^3 - 4(1)^2 + 10(1)\right] =$$

$$= \frac{16}{3}$$

This was created by Keenan Xavier Lee, 2013. See my website for more information, lee-apcalculus.weebly.com.

2nd Practice problem: $f(x) = \sqrt{x}$, $0 \le x \le 2$, find the area.



$$\int_0^2 [\sqrt{x}] - [0]\, dx$$

$$= \int_0^2 x^{1/2}\, dx$$

$$= \frac{x^{3/2}}{3/2} = \frac{2}{3}x^{3/2}\Big]_0^2$$

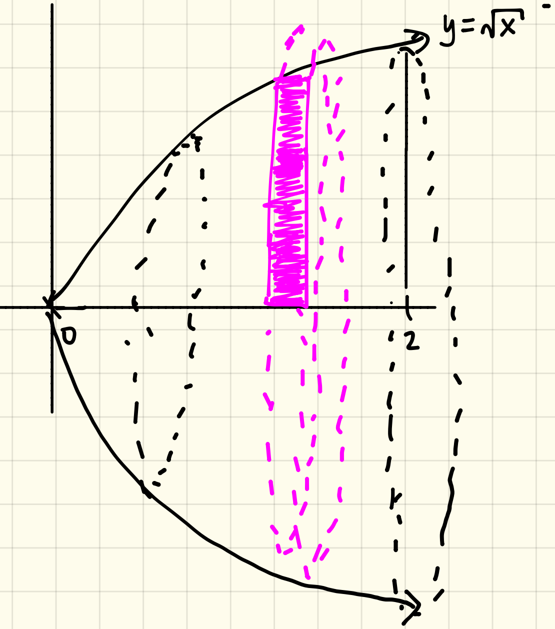
$$= \left[\frac{2}{3}(2)^{3/2}\right] - \left[\frac{2}{3}(0)^{3/3}\right]$$

$$= \frac{2}{3}\sqrt{2^3} - \frac{2}{3}\sqrt{0^3}$$

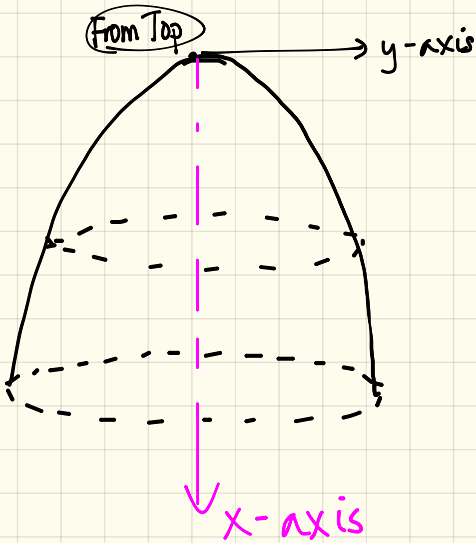
$$= \frac{2\sqrt{8}}{3} \approx 1.89$$

$\boxed{\text{new}}$ Volumes of Solids of Revolution (Disk Method)

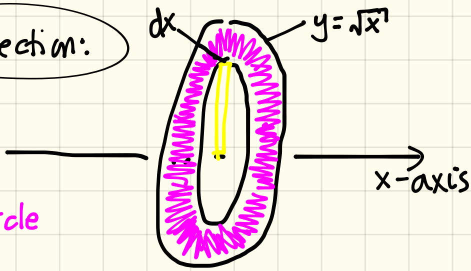
Now, let's consider the previous example where now we will find the Volume.



3D MODEL



A section:



Infinite amount of sections:

$$\int_0^2 \pi(\sqrt{x})^2\,dx = \int_0^2 \pi x\,dx$$

$$= \pi\int_0^2 x\,dx = \pi\left.\frac{x^2}{2}\right]_0^2$$

$$= \pi\left[\frac{(2)^2}{2}\right] - \left[\frac{(0)^2}{2}\right]$$

$$= 2\pi$$

Area of Circle

$= \pi r^2$

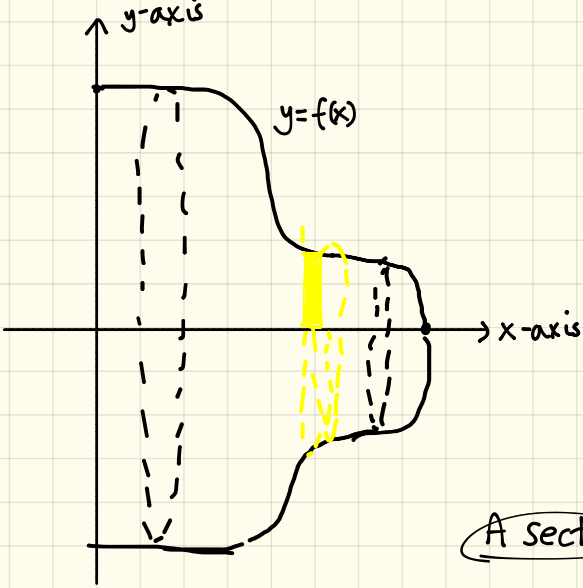
$= \pi(\sqrt{x})^2$

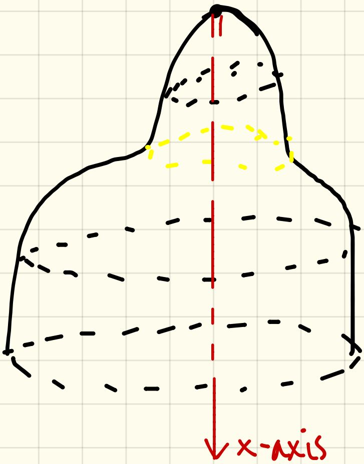
Volume of Circle

$= \text{Area} * dx$

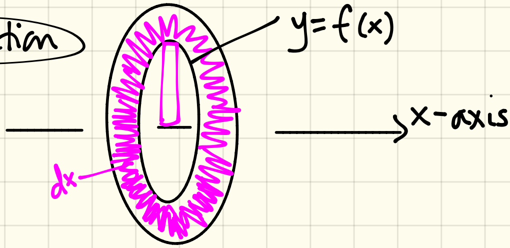
$= \pi(\sqrt{x})^2\,dx$

So let's generalize! Let's consider a curve $y=f(x)$ & find the volume revolving around x-axis.





A section



$A = \pi r^2 = \pi (f(x))^2$

$V = A * dx$

$= \pi (f(x))^2 dx$

"Take the sum of ALL OF THE DISKS"

$$\boxed{\text{Volume} = \int_a^b \pi (f(x))^2 \, dx}$$

– Formula when revolving around x-axis

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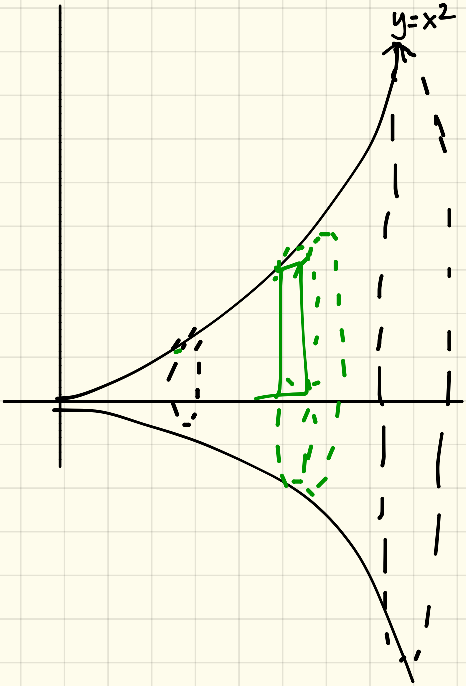
[Example 1] Find the volume of the solid by revolving around x-axis of $y=x^2$, $y=0$ and $x=2$.



$$V=\int_a^b \pi r^2\,dx = \int_0^2 \pi (x^2)^2\,dx$$

$$= \int_0^2 \pi x^4\,dx = \pi\int_0^2 x^4\,dx$$

$$= \pi \left.\frac{x^5}{5}\right]_0^2$$

$$= \pi\left[\frac{(2)^5}{5}\right] - \left[\frac{(0)^5}{5}\right]$$

$$= \frac{32\pi}{5}$$