8.4 Base e & Its Special Logarithm

$\boxed{\text{old}}$ Logarithms & their inverse

Recall:

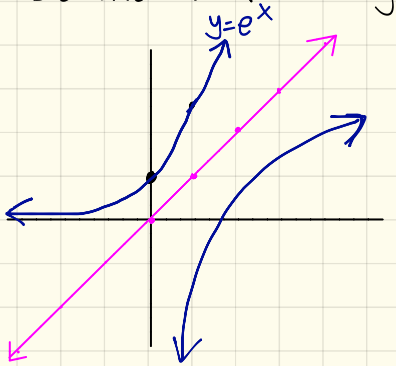
$\log_a a^x = x$, $a^{\log_a x} = x$.

$a^x = y \Longleftrightarrow \log_a y = x$

Change of base: $x = \dfrac{\log y}{\log a}$

$\boxed{\text{new}}$ Base e

Let's consider the function $y=e^x$. Graph the function and its inverse.



What would the inverse function be?

$x = \log_e y$

$$y = e^x \Longleftrightarrow x = \log_e y$$

The is a special name for $\log_e$ is ln.

$$y = e^x \Longleftrightarrow x = \ln(y)$$

Special facts:

$e^0 = 1$, $\ln(1) = 0$, $\ln(e^x) = x$, $e^{\ln x} = x$.

[Example] Find the unknown.

① $e^x = 45$

$x = \ln(45)$

$x \approx 3.807$

② $3e^{2x} - 4 = 44$

$3e^{2x} = 48$

$e^{2x} = 16$

$2x = \ln(16)$

$x = \dfrac{\ln(16)}{2}$

$x \approx 1.386$

③ $4 \ln x = -2$

$\ln x = -\frac{1}{2}$

$x = e^{-\frac{1}{2}}$

④ $-7 + \ln(2x) = 4$

$\ln(2x) = 11$

$2x = e^{11}$

$x = \dfrac{e^{11}}{2}$

⑤ $3 - 4\ln(8x+1) = 12$

$-4\ln(8x+1) = 9$

$\ln(8x+1) = \dfrac{-9}{4}$

$8x + 1 = e^{-\frac{9}{4}}$

$8x = e^{-\frac{9}{4}} - 1$

$x = \dfrac{e^{-\frac{9}{4}} - 1}{8}$

⑥ $e^{x+6} + 5 = 1$

$e^{x+6} = -4$

$x+6 = \ln(-4)$

$x = \ln(-4) - 6.$