### 8.4 Base e \& Its Special Logarithm

Old Logarithms \& their inverse
Recall:

$$
\log _{a} a^{x}=x, \quad a^{\log _{a} x}=x . \quad a^{x}=y \Leftrightarrow \log _{a} y=x
$$

change of base: $x=\frac{\log y}{\log a}$
new Base e
Let's consider the function $y=e^{x}$. Graph the function and its inverse.


What would the inverse function be?

$$
x=\log _{e} y
$$

$$
y=e^{x} \Longleftrightarrow x=\log _{e} y
$$

The is a special name for $\log _{e}$ is $\ln$.

$$
y=e^{x} \Longleftrightarrow x=\ln (y)
$$

Special facts:

$$
\begin{aligned}
& \text { Special facts: } \\
& e^{0}=1, \quad \ln (1)=0, \quad \ln \left(e^{x}\right)=x, \quad e^{\ln x}=x .
\end{aligned}
$$

This was created by Keenan Xavier Lee - 2014. See my website for more information, lee-apcalculus.weebly.com.
[Example] Find the unknown.
(1)

$$
\begin{aligned}
& e^{x}=45 \\
& x=\ln (45) \\
& x \approx 3.807
\end{aligned}
$$

(2)

$$
\begin{aligned}
3 e^{2 x}-4 & =44 \\
3 e^{2 x} & =48 \\
e^{2 x} & =16 \\
2 x & =\ln (16) \\
x & =\frac{\ln (16)}{2} \\
x & \approx 1.386
\end{aligned}
$$

(3)

$$
\begin{aligned}
4 \ln x & =-2 \\
\ln x & =-\frac{1}{2} \\
x & =e^{-\frac{1}{2}}
\end{aligned}
$$

$$
\begin{aligned}
-7+\ln (2 x) & =4 \\
\ln (2 x) & =11 \\
2 x & =e^{11} \\
x & =\frac{e^{\prime \prime}}{2}
\end{aligned}
$$

(b)

$$
\begin{aligned}
e^{x+6}+5 & =1 \\
e^{x+6} & =-4 \\
x+6 & =\ln (-4) \\
x & =\ln (-4)-6 .
\end{aligned}
$$

(5)

$$
\begin{aligned}
3-4 \ln (8 x+1) & =12 \\
-4 \ln (8 x+1) & =9 \\
\ln (8 x+1) & =\frac{-9}{4} \\
8 x+1 & =e^{-\frac{9}{4}} \\
8 x & =e^{-\frac{9}{4}}-1 \\
x & =\frac{e^{-\frac{9}{4}-1}}{8}
\end{aligned}
$$

