2.9 Trigonometric Ratios

Finding missing sides & missing angles using Right Triangle Trigonometry.

Standards:

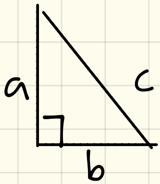
G.SRT.6

G.SRT.7

G.SRT.8

[Old] Pythagorean Theorem

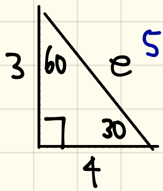
Recall: If there is a right triangle, then there is a relationship between the legs & hypotenuse which is $a^2+b^2=c^2$.



$$\boxed{a^2+b^2=c^2}$$

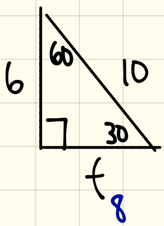
[Examples] Find the missing side.

①



$$a^2+b^2=c^2$$
$$(3)^2+(4)^2=e^2$$
$$9+16=e^2$$
$$25=e^2$$
$$\sqrt{25} \quad \sqrt{e^{\cancel{2}}}$$
$$5=e$$

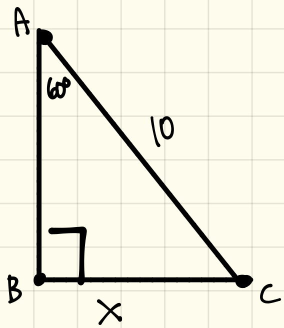
②



$$a^2+b^2=c^2$$
$$(6)^2+f^2=10^2$$
$$36+f^2=100$$
$$f^2=64$$
$$\sqrt{f^{\cancel{2}}}=\sqrt{64}$$
$$f=8$$

$\boxed{\text{new}}$ Trigonometric Ratios

Let's consider the below right triangle. Solve for the unknown.



We can't use Pythagorean Theorem because we have to use 2 legs to find the hypotenuse.

Dilemma: How do we find the unknown sides when only given one side in a right triangle?

- The way we can find unknown sides in a right is by using Trigonometric Ratios.

- The Trigonometric Functions we will be looking at are:

sine
↳ pronounced "sign"
↳ symbol sinθ

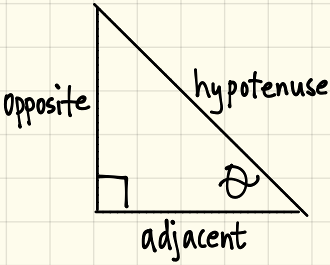
cosine
↳ pronounced "co-sign"
↳ symbol cosθ

tangent
↳ pronounced "tan-gent"
↳ tanθ

- Greek letter θ → pronounced "theta" representing an unknown angle. (like x represents an unknown number)

TRIG RATIOS

Memorize



$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

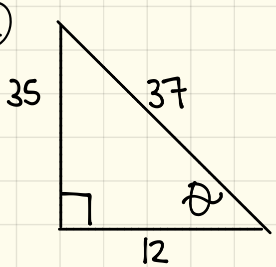
$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

- "opposite" means opposite side of θ
- "adjacent" means next to θ
- "hypotenuse" means longest or diagonal side.

A good way to memorize is SOH-CAH-TOA! Sounds like an indian word :)

[Examples] Find $\sin\theta$, $\cos\theta$ & $\tan\theta$ from the right triangles.

1.

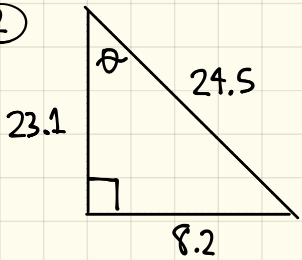


$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{35}{37}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{12}{37}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{35}{37}$$

(2)

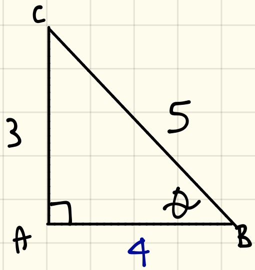


$\sin\theta = \frac{opp}{hyp} = \frac{8.2}{24.5}$

$\cos\theta = \frac{adj}{hyp} = \frac{23.1}{24.5}$

$\tan\theta = \frac{opp}{adj} = \frac{8.2}{23.1}$

(3) Consider ΔABC where $m\angle BAC = 90°$ and $\sin\theta = \frac{3}{5}$
Find $\overline{AB}$ and $\tan\theta$?



$\tan\theta = \frac{opposite}{adjacent} = \frac{3}{4}$

$a^2 + b^2 = c^2$

$3^2 + b^2 = 5^2$

$9 + b^2 = 25$

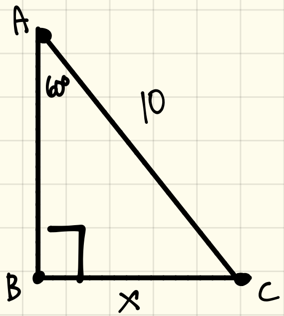
$b^2 = 16$

$\sqrt{b^2} = \sqrt{16}$

$b = 4.$

How do we use trig ratios to solve for missing sides?

Let's go back to the our dilemma.



- What's the angle of interest? 60°
- What's the position of side length x in respect to 60°? "opposite"
- What is the position of side length 60° in respect to 60°? "hypotenuse"

Which trig ratio is appropriate to use? $\sin\theta$.

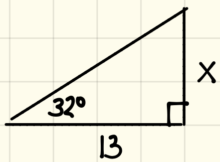
$\sin\theta = \frac{opp}{hyp} \Rightarrow \sin 60° = \frac{x}{10}$

Solve for x by cross multiplication.

$x = 10\sin 60°$
$x = 5\sqrt{3} \approx 8.66$

[More Examples] Find the missing side.

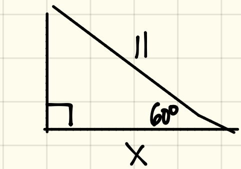
①



$\tan\theta = \frac{opp}{adj} \Rightarrow \tan 32° = \frac{x}{13}$

$13\tan 32° = x$
$8.12 \approx x$

②



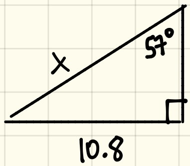
$$\cos\theta = \frac{adj}{hyp} \Rightarrow \cos 60^\circ = \frac{x}{11}$$

$$11\cos 60^\circ = x$$

$$\frac{11}{2} = x$$

$$5.5 = x$$

③

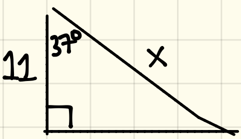


$$\sin\theta = \frac{opp}{adj} \Rightarrow \sin 57^\circ = \frac{10.8}{x}$$

$$\frac{x\cancel{\sin 57^\circ}}{\cancel{\sin 57}} = \frac{10.8}{\sin 57}$$

$$x \approx 12.88$$

④

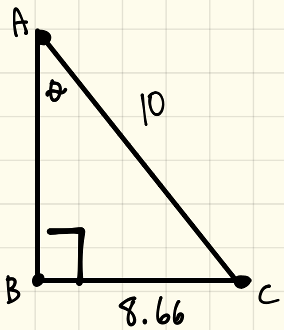


$$\cos\theta = \frac{adj}{hyp} \Rightarrow \cos 37^\circ = \frac{11}{x}$$

$$\frac{x\cancel{\cos 37^\circ}}{\cancel{\cos 37^\circ}} = \frac{11}{\cos 37^\circ}$$

$$x \approx 13.77$$

Let's consider a triangle with 2 given sides & a missing angle:



$$\sin\theta = \frac{opp}{adj} \Rightarrow$$

$$\sin\theta = \frac{8.66}{10}$$

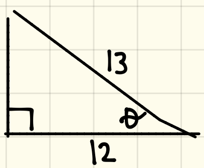
$$\sin^{-1}\left(\frac{8.66}{10}\right) = \theta$$

$$59.99^\circ \approx \theta$$
$$60^\circ \approx \theta$$

Now, I'm trying to find θ. ⇒ We need to isolate θ by using <u>INVERSE OPERATIONS</u>.

- The inverse of $\sin\theta$ is $\sin^{-1}\left(\frac{opp}{hyp}\right)$.
- The inverse of $\cos\theta$ is $\cos^{-1}\left(\frac{adj}{hyp}\right)$.
- The inverse of $\tan\theta$ is $\tan^{-1}\left(\frac{opp}{adj}\right)$

[More Examples] Find the missing angle.

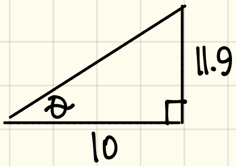
① 

$$\cos\theta = \frac{adj}{hyp} \Rightarrow \cos\theta = \frac{12}{13}$$

$$\cos^{-1}\left(\frac{12}{13}\right) = \theta$$

$$22.61^\circ \approx \theta$$
$$23^\circ \approx \theta$$

2

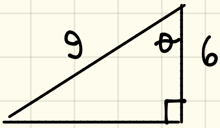


$$\tan\theta = \frac{opp}{adj} \Rightarrow \tan\theta = \frac{11.9}{10}$$

$$\tan^{-1}\left(\frac{11.9}{10}\right) = \theta$$

$$49.95^\circ \approx \theta$$
$$50^\circ \approx \theta$$

3



$$\cos\theta = \frac{adj}{hyp} \Rightarrow \cos\theta = \frac{6}{9}$$

$$\cos^{-1}\left(\frac{6}{9}\right) = \theta$$

$$48.2^\circ \approx \theta$$