A. Limits and Horizontal Asymptotes

What you are finding: You can be asked to find $\lim_{x\to a} f(x)$ or $\lim_{x\to\pm\infty} f(x)$. Typically, a horizontal asymptote (H.A.) problem is asking you find $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$.

How to find it: To find $\lim_{x\to a} f(x)$ algebraically, first determine if $f(a)$ exists and if so, $f(a)$ is $\lim_{x\to a} f(x)$. If not, express $f(x)$ as a fraction, factor both numerator and denominator if possible, do any cancellations and again, first determine if $f(a)$ exists and if so, $f(a)$ is $\lim_{x\to a} f(x)$. If $f(a)$ does not exist, then $\lim_{x\to a} f(x)$ may very well not exist. There are cases in which that limit very well may exist; for example $\lim_{x\to 0}\frac{\sin x}{x}$. Although not in the AB curriculum, it is recommended that students be familiar with L'Hopital's rule for such limits.

To find $\lim_{x\to\infty} f(x)$ or $\lim_{x\to-\infty} f(x)$, express $f(x)$ as a fraction. If both numerator and denominator are polynomials,

a) If the higher power of x is in the denominator, the H.A. is $y = 0$.
b) If both the numerator and denominator have the same highest power, the H.A. is the ratio of the coefficients of the highest power term in the numerator and the coefficient of the highest power term in the denominator.
c) If the higher power of x is in the numerator, there is no H.A.

When the numerator or denominator does not contain polynomials, students should see the effect of plugging in very large or very small numbers.

Note: L'Hopital's rule can be used to find these limits if students have learned it. L'Hopital's rule is not in the AB curriculum. I recommend teaching it at the end of the year if there is time.

1. Find $\lim_{x\to 0}\frac{6x^5-8x^3}{9x^3-6x^5}$

A. $\frac{2}{3}$ B. $\frac{-8}{9}$ C. $\frac{4}{3}$ D. $\frac{-8}{3}$ E. nonexistent

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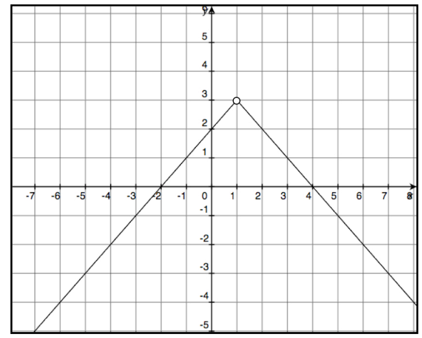
2. Find $\lim_{x\to-\infty}\frac{(3x-1)(x^2-4)}{(2x+1)^2(x-1)}$

A. $-\frac{3}{2}$ B. $\frac{3}{2}$ C. $\frac{3}{4}$ D. 1 E. ∞

3. The figure to the right shows the graph of $f(x)$.
Which of the following statements are true?



I. $\lim_{x \to 1^-} f(x)$ exists

II. $\lim_{x \to 1^+} f(x)$ exists

III. $\lim_{x \to 1} f(x)$ exists

A. I only B. II only C. I and II only D. I, II and III E. none are true

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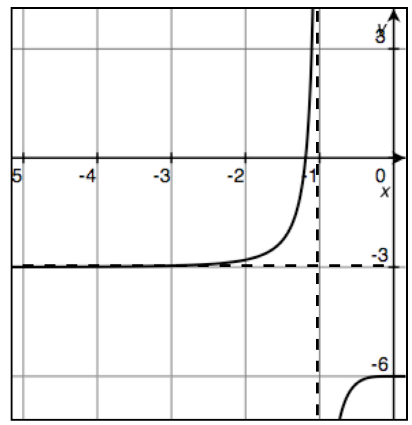
4. The function f is given by $f(x) = \dfrac{ax^4 + 6}{x^4 + b}$. The figure to the right shows a portion of the graph of f. Which of the following could be the values of the constants a and b?



A. $a = -3,\ b = -1$
B. $a = 3,\ b = 1$
C. $a = 3,\ b = -1$
D. $a = -3,\ b = 1$
E. $a = 6,\ b = -1$

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5. If $a \neq 0$ and n is a positive integer, then $\lim_{x \to a} \dfrac{x^n - a^n}{x^{2n} - a^{2n}}$ is

A. $\dfrac{1}{a^n}$ B. $\dfrac{1}{2a^n}$ C. $\dfrac{1}{a^{2n}}$ D. 0 E. nonexistent

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6. What are all the horizontal asymptotes of $f(x) = \dfrac{6 + 3e^x}{3 - e^x}$ in the xy-plane?

A. $y = 3$ only B. $y = -3$ only C. $y = 2$ only D. $y = -3$ and $y = 0$ E. $y = -3$ and $y = 2$

B. Definition of Derivative

What you are finding: The derivative of a function is a formula for the slope of the tangent line to the graph of that function. There are two definitions that are commonly used that students should know.

How to find it: $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ or $f'(x)=\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$. Students need to know that differentiable functions at a point (derivative exists at the point) are necessarily continuous but continuous functions are not necessarily differentiable. These types of problems usually necessitate that students recognize these limits as a derivative and use derivative rules to calculate it.

7. $\lim_{h\to 0}\frac{\sin(\pi+h)-\sin\pi}{h}=$

A. 0 B. $\cos x$ C. -1 D. π E. 1

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8. Let f be a function such that $\lim_{x\to 4}\frac{f(x)-f(4)}{x-4}=2$. Which of the following must be true?

I. f is continuous at $x = 4$.

II. f is differentiable at $x = 4$.

III. The derivative of f' is continuous at $x = 4$.

A. I only B. II only C. I and II only D. I and III only E. I, II and III

9. If $f(x) = \tan^{-1}(2x)$, find $\lim\limits_{x \to 2} \dfrac{f(x) - f(2)}{x - 2}$.

A. $\dfrac{2}{17}$ B. $\dfrac{1}{17}$ C. $\dfrac{-2}{15}$ D. $\dfrac{-1}{15}$ E. nonexistent

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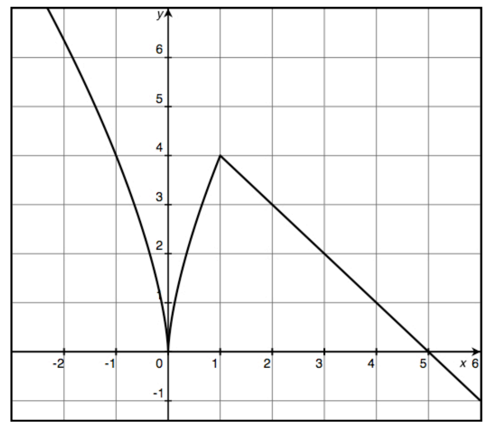
10. The graph of $f(x)$ consists of a curve that is symmetric to the y-axis on [-1, 1] and a line segment as shown to the right. Which of the following statements about f is false?



A. $\lim\limits_{x \to 0} \left[f(x) - f(0) \right] = 0$

B. $\lim\limits_{x \to 0} \dfrac{f(x) - f(0)}{x} = 0$

C. $\lim\limits_{h \to 0} \dfrac{f(0+h) - f(0-h)}{2h} = 0$

D. $\lim\limits_{x \to 2} \dfrac{f(x) - f(2)}{x - 2} = -1$

E. $\lim\limits_{h \to 0} \dfrac{f(1+h) - f(1)}{h}$ does not exist

C. Taking Derivatives with Basic Functions

What you are finding: The derivative of a function is a formula for the slope of the tangent line to the graph of that function. Students are required to know how to take derivative of basic functions, trig functions, logarithmic and exponentials, and inverse trig functions. They also need to be able to take derivative of inverse functions. Finally, rules such as power, product, quotient, chain rule, and taking derivatives implicitly must be a process that students have down perfectly.

How to find it:

Power Rule: $\frac{d}{dx}\left(x^n\right) = nx^{n-1}$

Product Rule: $\frac{d}{dx}\left[f(x)\cdot g(x)\right] = f(x)\cdot g'(x) + g(x)\cdot f'(x)$

Quotient Rule: $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\cdot f'(x) - f(x)\cdot g'(x)}{\left[g(x)\right]^2}$

Chain Rule: $\frac{d}{dx}\left[f\left(g(x)\right)\right] = f'\left[g(x)\right]\cdot g'(x)$

$\frac{d}{dx}(\sin x) = \cos x$ $\quad \frac{d}{dx}(\cos x) = -\sin x$ $\quad \frac{d}{dx}(\tan x) = \sec^2 x$ $\quad \frac{d}{dx}(\csc x) = -\csc x\cot x$ $\quad \frac{d}{dx}(\sec x) = \sec x\tan x$ $\quad \frac{d}{dx}(\cot x) = -\csc^2 x$

$\frac{d}{dx}(\ln x) = \frac{1}{x}$ $\quad \frac{d}{dx}\left(e^x\right) = e^x$ $\quad \frac{d}{dx}\left(a^x\right) = a^x \ln a$ $\quad \frac{d}{dx}\left(\sin^{-1} x\right) = \frac{1}{\sqrt{1-x^2}}$ $\quad \frac{d}{dx}\left(\cos^{-1} x\right) = \frac{-1}{\sqrt{1-x^2}}$ $\quad \frac{d}{dx}\left(\tan^{-1} x\right) = \frac{1}{1+x^2}$

11. If $f(x) = (2x+1)\left(x^2-3\right)^4$, then $f'(x) =$

A. $2\left(x^2-3\right)^3\left(x^2+4x-1\right)$ B. $4(2x+1)\left(x^2-3\right)^3$ C. $8x(2x+1)\left(x^2-3\right)^3$
D. $2\left(x^2-3\right)^3\left(3x^2+x-3\right)$ E. $2\left(x^2-3\right)^3\left(9x^2+4x-3\right)$

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12. If $f(x) = \ln\left(\frac{e}{x^n}\right)$, and n is a constant, then $f'(x) =$

A. $\frac{-n}{x}$ B. $\frac{x^n}{e}$ C. $\frac{-1}{x^n}$ D. $\frac{e}{x^n}$ E. 0

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13. If $f(x) = \sqrt[3]{\cos^2(3x)}$, then $f'(x) =$

A. $\frac{-2}{3\sqrt[3]{\sin 3x}}$ B. $\frac{-2}{\sqrt[3]{\sin 3x}}$ C. $\frac{-2\sin 3x}{\sqrt[3]{\cos 3x}}$ D. $\frac{2}{\sqrt[3]{\cos 3x}}$ E. $\frac{-2\sin 3x}{3\sqrt[3]{\cos 3x}}$

14. The table below gives value of the differentiable function f and g at $x=-1$. If $h(x)=\dfrac{f(x)-g(x)}{2f(x)}$, then $h'(-1)=$

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
-1	-2	4	e	-3

A. $\dfrac{-e-3}{4}$ B. $\dfrac{e+3}{2e}$ C. $\dfrac{e-6}{8}$ D. $\dfrac{2e-3}{4}$ E. $\dfrac{-4e-3}{4}$

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15. The functions f and g are differentiable and $f(g(x))=x^2$ for all x. If $f(4)=8,\ g(4)=8,\ f'(8)=-2$, what is the value of $g'(4)$?

A. $\dfrac{-1}{8}$ B. $\dfrac{-1}{2}$ C. -2 D. -4 E. Insufficient data

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16. $f(x)=e^{2\ln\tan(x^2)}$, then $f'(x)=$

A. $e^{2\ln\tan(x^2)}$ B. $\dfrac{e^{2\ln\tan(x^2)}}{x^2}$ C. $\dfrac{e^{2\ln\tan(x^2)}}{2\tan x^2}$ D. $\dfrac{2\sin(x^2)}{\cos^3(x^2)}$ E. $\dfrac{4x\sin(x^2)}{\cos^3(x^2)}$

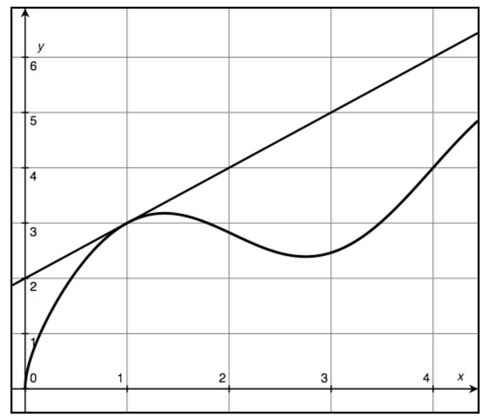
D. Tangent Lines and Local Linear Approximations

What you are finding: You typically have a function f and you are given a point on the function. You want to find the equation of the tangent line to the curve at that point.

How to find it: You use your point-slope equation: $y - y_1 = m(x - x_1)$ where m is the slope and (x_1, y_1) is the point. Typically, to find the slope, you take the derivative of the function at the specified point: $f'(x_1)$.

What you are finding: You typically have a function f given as a set of points as well as the derivative of the function at those x-values. You want to find the equation of the tangent line to the curve at a value c close to one of the given x-values. You will use that equation to approximate the y-value at c. This uses the concept of local linearity – the closer you get to a point on a curve, the more the curve looks like a line.

How to find it: You use your point-slope equation: $y - y_1 = m(x - x_1)$ where m is the slope and (x_1, y_1) is the point closest to c. Typically, to find the slope, you take the derivative $f'(x_1)$ of the function at the closest x-value given. You then plug c into the equation of the line. Realize that it is an approximation of the corresponding y-value. If 2nd derivative values are given as well, it is possible to determine whether the approximated y-value is above or below the actual y-value by looking at concavity. For instance, if we wanted to approximate $f(1.1)$ for the curve in the graph to the right, we could find $f'(x)$, use it to determine the equation of the tangent line to the curve at $x = 1$ and then plug 1.1 into that linear equation. If information were given that the curve was concave down, we would know that the estimation over-approximated the actual y-value.



17. If $f(x) = \sin^3(4x)$, find $f'\left(\frac{\pi}{3}\right)$.

A. $-\frac{3}{2}$ B. $-\frac{9}{2}$ C. $-3\sqrt{3}$ D. $3\sqrt{3}$ E. $-\frac{9}{8}$

18. If $f(x) = 2^{x^2 - x - 1}$, find the equation of the tangent line to f at $x = -1$.

A. $y = 2x + 4$
B. $y = -6x - 4$
C. $y = 2 - 2\ln 2(x+1)$
D. $y = 2 - 6\ln 2(x+1)$
E. $y = 2 - 3\ln 2(x+1)$

19. What is the slope of the line tangent to the graph of $y = \dfrac{1}{e^{2x}(x-2)}$ at $x = 1$?

A. $\dfrac{-2}{e}$ B. $\dfrac{-1}{e^2}$ C. 0 D. $\dfrac{1}{e^2}$ E. $\dfrac{-2}{e^2}$

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20. What is the equation of the line normal to the graph of $y = 2\sin x \cos x - \sin x$ at $x = \dfrac{3\pi}{2}$?

A. $y = 2x - 3\pi + 1$ B. $y = -2x + 3\pi + 1$ C. $y = \dfrac{x}{2} - \dfrac{3\pi}{4} + 1$

D. $y = -\dfrac{x}{2} + \dfrac{3\pi}{4} + 1$ E. $y = -\dfrac{x}{2} + \dfrac{3\pi}{4} - 1$

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21. **(Calc)** Let f be the function given by $f(x) = \dfrac{4x^2}{e^x - e^{-x}}$. For what positive value(s) of c is $f'(c) = 1$?

A. 1.122 B. 0.824 and 4.306 C. 0.258 and 3.260
D. 3.264 E. 0.523 and 4.307

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22. The function f is twice differentiable with $f(-3) = -2$ and $f'(-3) = -4$. For what value of c is the approximation of $f(c)$ using the tangent line of f at $x = c$ equal to c?

A. –4.667 B. –0.4 C. −2.8 D. −2.5 E. −3.333

23. The line $x + y = k$, where k is a constant, is tangent to the graph of $y = 2x^3 - 9x^2 - x + 1$. What are the only possible values of k?

A. 1 only B. 0 and - 29 C. 1 and -29 D. 0 and 3 E. 1 and -26

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24. For the function f, $f'(x) = 4x - 3$ and $f(2) = 4$. What is the approximation for $f(2.1)$ found by using the tangent line to the graph of f at $x = 2$.

A. -2.6 B. 4.5 C. 4.8 D. 5.4 E. 9.4

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25. **(Calc)** The function f is defined for $x > 0$ with $f(\pi) = 1$, and $f'(x) = \sin\left(\frac{1}{x}\right)$. Use the line tangent to f at $x = \pi$ to approximate the value of $f(4)$ and whether it over or under-approximates $f(4)$.

A. 1.269 – under-estimates B. 1.269 – over-estimates C. 1.815 – under-estimates
D. 1.815 – over-estimates E. 0.731 – over-estimates

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26. The number of vehicles waiting to get past an accident scene is modeled by a twice-differentiable N of t, where t is measured in minutes. For $0 < t < 8$, $N''(t) > 0$. The table below gives selected values of the rate of change $N'(t)$ over the time interval $0 \le t \le 8$. The number of vehicles in line at $t = 4$ is 45. By using the tangent line approximation at $t = 4$, which of the following is a possible value for the number of vehicles in line at $t = 4.5$ minutes?

t (minutes)	0	1	4	6	8
$N'(t)$ (vehicles per minute)	7	10	24	32	42

I. 57 II. 59 III. 63

A. I only B. II only C. III only D. II and III only E. I, II and III

E. Implicit Differentiation

What you are finding: You are asked to either find $\frac{dy}{dx}$ or $\frac{dy}{dx}$ at some point (x,y). You perform implicit differentiation when the function is not given explicitly in the form "$y =$" or when it is difficult or impossible to put it in that form.

How to find it: Take the derivative of every term in the given expression, remembering that the chain rule is used and the derivative of y is not 1 but $\frac{dy}{dx}$. If you are asked to find $\frac{dy}{dx}$ at a given point, it is best to plug the point in, once you have taken the derivative and solve for $\frac{dy}{dx}$. If the $\frac{dy}{dx}$ cancels out of the equation, the tangent line is vertical at that point. Implicit examples show up in related rates problems when you are doing implicit differentiation with respect for time *t*.

27. If $\sin(\pi x)\cos(\pi y) = y$, find $\frac{dy}{dx}$ at $(1,0)$

A. $1+\pi$ B. $1-\pi$ C. -1 D. $-\pi$ E. π

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28. Find the equation of the tangent line to $x^3 + y^3 = 3xy + 5x - 3y$ at $(2,2)$.

A. $x+9y=20$ B. $13x-3y=20$ C. $9y-x=16$
D. $y=\frac{-1}{9}$ E. $y=2$

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29. If $x^2 + 2y^2 = 34$, find the behavior of the curve at (-4, 3).

A. Increasing, concave up B. Increasing, concave down C. Decreasing, concave up
D. Decreasing, concave down E. Decreasing, inflection point

30. Find the y-intercept of the tangent line to $4\sqrt{x}+2\sqrt{y}=x+y+3$ at the point (9, 4).

A. -2 B. 10 C. $\frac{5}{2}$ D. 15 E. $\frac{4}{9}$

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31. If $x^2+y^2=a^2$, where a is a constant, find $\frac{d^2y}{dx^2}$.

A. $\frac{-y^2-x^2}{y^2}$ B. $\frac{x^2-y^2}{y^3}$ C. $\frac{-1}{y^2}$ D. $\frac{-a^2}{y^3}$ E. $\frac{1}{a^2}$

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32. Let x and y be functions of time t that are related by the equation $y^2=xy+e$. At time $t=1$, the value of y is e and $\frac{dy}{dt}=2$. Find $\frac{dx}{dt}$ when $t=1$.

A. $\frac{2e-2}{e}$ B. $\frac{2e+2}{e}$ C. $\frac{2}{e}$ D. $\frac{e+2}{e}$ E. $\frac{e-2}{e}$

F. Derivatives of Inverse Functions

What you are finding: There is probably no topic that confuses students (and teachers) more than inverses. The inverse of a function *f* is another function f^{-1} that "undoes" what *f* does. So $f^{-1}(f(x)) = x$. For instance, the inverse of adding 5 is subtracting 5. Start with any number *x*, add 5, then subtract 5, and you are back to *x*. Do not confuse the inverse f^{-1} with the reciprocal. $x^{-1} = \frac{1}{x}$ but $(f(x))^{-1} \neq \frac{1}{f(x)}$.

To find the inverse of a function, you replace *x* with *y* and *y* with *x*. The inverse to the function $y = 4x - 1$ is $x = 4y - 1$ or $y = \frac{x+1}{4}$. In this section, you are concerned with finding the derivative of the inverse to a function: $\frac{d}{dx}\left[f(x)\right]^{-1}$.

How to find it: The formula used is: $\frac{dy}{dx} = \frac{1}{f'(y)}$. But what I suggest, rather than memorizing this formula, is to switch *x* and *y* to find the inverse, and then take the derivative, using implicit differentiation: $x = f(y) \Rightarrow 1 = f'(y)\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{1}{f'(y)}$. Students should also know the formulas for derivatives of inverse trig functions and how they are found: $\frac{d}{dx}\left(\sin^{-1} x\right) = \frac{1}{\sqrt{1-x^2}}$ $\frac{d}{dx}\left(\cos^{-1} x\right) = \frac{-1}{\sqrt{1-x^2}}$ $\frac{d}{dx}\left(\tan^{-1} x\right) = \frac{1}{1+x^2}$

33. If $f(x) = x^3 + x^2 + x + 1$ and $g(x) = f^{-1}(x)$, what is the value of $g'(4)$?

A. $\frac{1}{85}$ B. 1 C. $\frac{1}{57}$ D. $\frac{1}{6}$ E. 24

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34. Let *f* be a differentiable function such that $f(-4) = 12$, $f(9) = -4$, $f'(4) = -6$, $f'(9) = 3$. The function *g* is differentiable and $g(x) = f^{-1}(x)$ for all *x*. What is the value of $g'(-4)$?

A. $\frac{-1}{6}$ B. $\frac{-1}{4}$ C. $\frac{1}{3}$ D. $\frac{1}{9}$ E. Insufficient data

35. The function g is differentiable for all real numbers. The table below gives values of the function and its first derivatives at selected values of x. If g^{-1} is the inverse function of g, what is the equation for the line tangent to the graph of $y = g^{-1}(x)$ at $x = 4$?

x	$g(x)$	$g'(x)$
-2	4	2
4	-3	5

A. $y+3=\frac{1}{5}(x-4)$

B. $y+2=\frac{1}{5}(x-4)$

C. $y+2=2(x-4)$

D. $y+3=\frac{1}{2}(x-4)$

E. $y+2=\frac{1}{2}(x-4)$

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36. **(Calc)** Find the derivative of $f^{-1}(x)$ for $f(x)=x^3-3x^2-2x+1$ at $x = 2$.

A. 0.064

B. –0.500

C. 3.627

D. 0.005

E. -0.045

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37. If $y=\csc^{-1}(x^2)$, which of the following represents $\frac{dy}{dx}$?

A. $2x\sin x^2$

B. $-2x\sin y\tan y$

C. $\frac{2x}{\cos y}$

D. $\frac{-2x}{\sin^2 y}$

E. $\frac{-2x}{\sin^2(x^2)}$

G. Continuity and Differentiability

What you are finding: Typical problems ask students to determine whether a function is continuous and/or differentiable at a point. Most functions that are given are continuous in their domain, and functions that are not continuous are not differentiable. So functions given usually tend to be piecewise and the question is whether the function is continuous and also differentiable at the *x*-value where the function changes from one piece to the other.

How to find it: Continuity: I like to think of continuity as being able to draw the function without picking your pencil up from the paper. But to prove continuity at $x = c$, you have to show that $\lim_{x \to c} f(x) = f(c)$.
Usually you will have to show that $\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x)$.
Differentiability: I like to think of differentiability as "smooth." At the value *c*, where the piecewise function changes, the transition from one curve to another must be a smooth one. Sharp corners (like an absolute value curve) or cusp points mean the function is not differentiable there. The test for differentiability at $x = c$ is to show that $\lim_{x \to c^-} f'(x) = \lim_{x \to c^+} f'(x)$. So if you are given a piecewise function, check first for continuity at $x = c$, and if it is continuous, take the derivative of each piece, and check that the derivative is continuous at $x = c$. Lines, polynomials, exponentials, sine and cosines curves are differentiable everywhere.

38. Let f be the function defined below, where *c* and *d* are constants. If f is differentiable at $x = -1$, what is the value of $c - d$?

$$f(x) = \begin{cases} x^2 + (2c+1)x - d, x \geq -1 \\ e^{2x+2} + cx + 3d, x < -1 \end{cases}$$

A. -2 B. 0 C. 2 D. 3 E. 4

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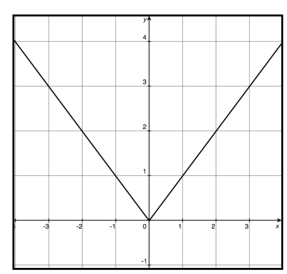
39. The graph of $f(x) = \sqrt{x^2 + 0.0001} - 0.01$ is shown in the graph to the right. Which of the following statements are true?



I. $\lim_{x \to 0} f(x) = 0$.
II. f is continuous at $x = 0$.
III. f is differentiable at $x = 0$.

A. I only B. II only C. I and II only D. I, II, and III E. None are true

40. Let $f(x)$ be given by the function below. What values of *a*, *b*, and *c* do **NOT** make $f(x)$ differentiable?

$$f(x)=\begin{cases} a\cos x, x \le 0 \\ b\sin(x+c\pi), x>0 \end{cases}$$

A. $a=0,\ b=0,\ c=0$ B. $a=0,\ b=0,\ c=100$ C. $a=5,\ b=5,\ c=0.5$

D. $a=-1,\ b=1,\ c=1$ E. $a=-8,\ b=8,\ c=-2.5$

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41. Let f be the function defined below. Which of the following statements about f are **NOT** true?

$$f(x)=\begin{cases} \dfrac{x^3-1}{x-1}, x \ne 1 \\ 3x, x=1 \end{cases}$$

I. f has a limit at $x=1$.
II. f is continuous at $x=1$.
III. f is differentiable at $x=1$.
IV. The derivative of f' is continuous at $x=1$.

A. IV only B. III and IV only C. II, III, and IV only
D. I, II, III, and IV E. All statements are true

H. Intermediate Value and Mean Value Theorem (MVT)

What it says: (IVT) If you have a continuous function on $[a,b]$ and $f(b) \neq f(a)$, the function must take on every value between $f(a)$ and $f(b)$ at some point between $x = a$ and $x = b$. For instance, if you are on a road traveling at 40 mph and a minute later you are traveling at 50 mph, at some time within that minute, you must have been traveling at 41 mph, 42 mph, and every possible value between 40 mph and 50 mph.

What it says: (MVT) If $f(x)$ is continuous on $[a,b]$ and differentiable on (a, b), there must be some value of c between a and b such that $f'(c) = \dfrac{f(b) - f(a)}{b - a}$. In words, this says that there must be some value between a and b such that that the tangent line to the function at that value is parallel to the secant line between a and b.

42. The function f is continuous and non-linear for $-3 \leq x \leq 7$ and $f(-3) = 5$ and $f(7) = -5$. If there is no value c, where $-3 < c < 7$, for which $f'(c) = -1$, which of the following statements must be true?

A. For all k, where $-3 < k < 7$, $f'(k) < -1$.
B. For all k, where $-3 < k < 7$, $f'(k) > -1$.
C. For some k, where $-3 < k < 7$, $f'(k) = 0$.
D. For $-3 < k < 7, f'(k)$ exists.
E. For some k, where $-3 < k < 7$, $f'(k)$ does not exist.

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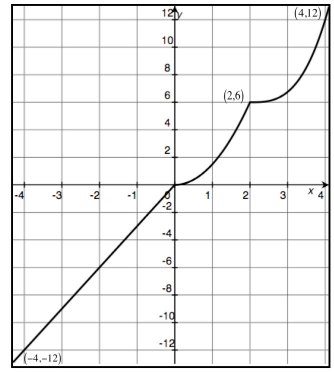
43. A new robotic dog called the IPup went on sale at (9 AM) and sold out within 8 hours. The number of customers in line to purchase the IPup at time t *is* modeled by a differentiable function A where $0 \leq t \leq 8$. Values of $A(t)$ are shown in the table below. For $0 \leq t \leq 8$, what is the fewest number of times at which $A'(t) = 0$?

t (hours)	0	1	2	3	4	5	6	7	8
$A(t)$ people	150	185	135	120	75	75	100	120	60

A. 0 B. 2 C. 3 D. 4 E. 5

44. A continuous function f is defined on the closed interval $-4 \le x \le 4$. The graph of the function, shown in the figure to the right consists of a line and two curves. There is a value a, $-4 \le a < 4$, for which the Mean Value Theorem, applied to the interval $[a, 4]$ guarantees a value c, $a \le c < 4$ at which $f'(c) = 3$. What are possible values of a?



I. -4   II. 0   III. 2

A I only   B. II only   C. III only
D. II and III only   E. I, II, and III

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45. Let f be a twice-differentiable function such that $f(a) = b$ and $f(b) = a$ for two unknown constants a and b, $a < b$. Let $g(x) = f(f(x))$. The Mean Value Theorem applied to g' on $[a, b]$ guarantees a value k such that $a < k < b$ such that

A. $g'(k) = 0$ B. $g''(k) = 0$ C. $g'(k) = 1$ D. $g''(k) = 1$ E. $g'(k) = b - a$

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46. **(Calc)** There are value(s) of c that satisfy the Mean-Value Theorem for $f(x) = 2\cos x - 4\cos 2x$ on $[0, \pi]$. Find the sum of these values.

A. 1.455   B. 4.493   C. 4.596   D. 1.687   E. -1.273

I. Horizontal and Vertical Tangent Lines

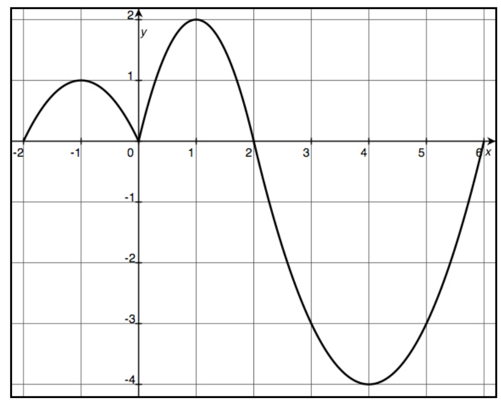
How to find them: You need to work with $f'(x)$, the derivative of function *f*. Express $f'(x)$ as a fraction.

Horizontal tangent lines: set $f'(x)=0$ and solve for values of *x* in the domain of *f*.

Vertical tangent lines: find values of *x* where $f'(x)$ is undefined (the denominator of $f'(x)=0$).

In both cases, to find the point of tangency, plug in the *x* values you found back into the function *f*. However, if both the numerator and denominator of $f'(x)$ are simultaneously zero, no conclusion can be made about tangent lines. These types of problems go well with implicit differentiation.

47. The graph of f', the derivative of f is shown to the right for $-2 \le x < 6$. At what values of *x* does *f* have a horizontal tangent line?



A. $x = 0$ only
B. $x = -1, x = 1, x = 4$
C. $x = 2$ only
D. $x = -2, x = 2, x = 6$
E. $x = -2, x = 0, x = 2, x = 6$

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48. At which points is the tangent line to the curve $8x^2 + 2y^2 = 6xy + 14$ vertical?

I. (-2, -3) II (3, 8) III. (4, 6)

A. I only B. II only C. III only D. I and II only E. I and III only

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49. How many horizontal (H) and vertical (V) tangent lines does the graph of $\left(x^2+4\right)y = 8$ have?

A. 2 H, 1V B. 2 H, 0V C. 1 H, 0V D. 0 H, 2V E. 0 H, 0V

J. Related Rates

This is a difficult topic to represent. There are many types of questions that could be interpreted as related rates problems. It can be argued that straight-line motion problems fall under the category of related rates. So do the types of questions that interpret a derivative as a rate of change. In this section, we will only focus on questions that use geometric shapes with different quantities changing values.

What you are finding: In related rates problems, any quantities, given or asked to calculate, that are changing are derivatives with respect to time. If you are asked to find how a volume is changing, you are being asked for $\frac{dV}{dt}$. Typically in related rates problems, you need to write the formula for a quantity in terms of a single variable and differentiate that equation with respect to time, remembering that you are actually performing implicit differentiation. If the right side of the formula contains several variables, typically you need to link them somehow. Constants may be plugged in before the differentiation process but variables that are changing may only be plugged in after the differentiation process.

50. A tube is in the shape of a right circular cylinder. The height is increasing at the rate of 1.5 inches/sec while the radius is decreasing that the rate of 0.5 inches/sec. At the time that the radius is 4 inches and the length is 14 inches, how is the lateral surface area of the tube changing? (The lateral surface area of a right circular cylinder with radius r and height h is $2\pi rh$).

A. Decreasing at $1\ \text{in}^2/\text{sec}$ B. Increasing at $1\ \text{in}^2/\text{sec}$ C. Decreasing at $2\pi\ \text{in}^2/\text{sec}$
D. Increasing at $26\pi\ \text{in}^2/\text{sec}$ E. Increasing at $38\pi\ \text{in}^2/\text{sec}$

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51. A sphere with radius r has volume $\frac{4}{3}\pi r^3$ and surface area $4\pi r^2$. At a certain instant the rate of decrease of its volume is twice the rate of decrease of its surface area. What is the radius of the sphere at that point in time?

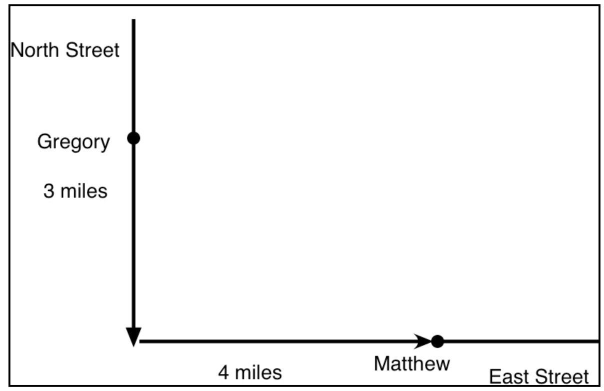
A. $\frac{1}{4}$ B. $\frac{1}{2}$ C. 1 D. 2 E. 4

---

52. A large block of ice in the shape of a cube is melting. All sides of the cube melt at the same rate. At the time that the block is s feet on each side, its surface area is decreasing at the rate of $24\ \text{ft}^2/\text{hr}$. At what rate is the volume of the block decreasing at that time?

A. $12s\ \text{ft}^3/\text{hr}$ B. $6s\ \text{ft}^3/\text{hr}$ C. $4s\ \text{ft}^3/\text{hr}$ D. $2s\ \text{ft}^3/\text{hr}$ E. $s\ \text{ft}^3/\text{hr}$

53. Matthew is visiting Gregory at his home on North Street. Shortly after Matthew leaves, Gregory realizes that Matthew left his wallet and begins to chase him. When Gregory is 3 miles from the $90°$ intersection along North Street traveling at 40 mph towards the intersection, Matthew is 4 miles along East street traveling away from the intersection at 25 mph. At that time, how fast is the distance between the two men changing?



A. getting closer at 4 mph
B. getting further away at 44 mph
C. getting closer at 44 mph
D. Getting closer at 5 mph
E. getting closer at 7 mph

---

54. A right triangle has legs a and b and hypotenuse c. The lengths of leg a and leg b are changing but at a certain instant, the area of the triangle is not changing. Which statement must be true?

A. $a = b$
B. $\frac{da}{dt} = \frac{db}{dt}$
C. $\frac{da}{dt} = -\frac{db}{dt}$
D. $a\frac{da}{dt} = -b\frac{db}{dt}$
E. $a\frac{db}{dt} = -b\frac{da}{dt}$

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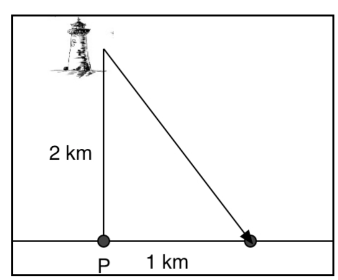
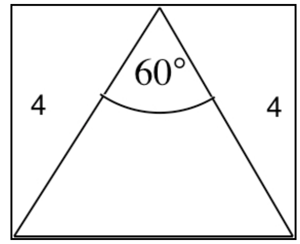
55. A lighthouse is 2 km from a point P along a straight shoreline and its light makes 1 revolution per minute. How fast, in km/min, is the beam of light moving along the shoreline when it is 1 km from point P?



A. 2.5
B. 10π
C. 20π
D. 5π
E. 10

56. **(Calc)** An isosceles triangle with its two equal sides of 4 inches has the angle between them changing at the rate of $10°/\text{min}$. How fast is the 3rd side of the triangle changing, in inches/min, when the angle between the two given sides is $60°$?



A. 0.605 B. 17.321 C. 34.641
D. 1.209 E. 8.660

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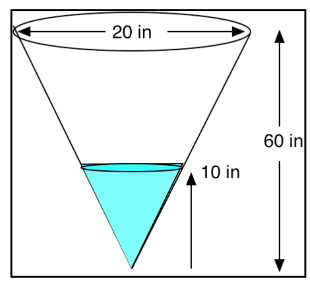
57. **(Calc)** A conical tank is leaking water at the rate of $75\,\text{in}^3/\text{min}$. At the same time, water is being pumped into the tank at a constant rate. The tank's height is 60 in while its top diameter is 20 inches. If the water level is rising at the rate of $5\,\text{in}/\text{min}$ when the height of the water is 10 inches high, find the rate in which water is being pumped into the tank to the nearest in^3/min. (The volume of a cone is given by $V = \frac{1}{3}\pi r^2 h$)



A. 44 in^3/min B. 175 in^3/min C. 250 in^3/min D. 119 in^3/min E. 31 in^3/min

K. Straight-Line Motion – Derivatives

What you are finding: Typically in these problems, you work your way from the position function $x(t)$ to the velocity function $v(t)$ to the acceleration function $a(t)$ by the derivative process. Finding when a particle is stopped involves setting the velocity function $v(t) = 0$. Speed is the absolute value of velocity. Determining whether a particle is speeding up or slowing down involves finding both the velocity and acceleration at a given time. If $v(t)$ and $a(t)$ have the same sign, the particle is speeding up at a specific value of *t*. If their signs are different, then the particle is slowing down.

58. A particle moving along the *x*-axis such that at any time $t \geq 0$ its position is given by $x(t) = 2 - 30t + 81t^2 - 20t^3$. For what values of *t* is the particle moving to the left?

A. $0 \leq t < \frac{27}{20}$ B. $t > \frac{27}{20}$ C. $0 \leq t < \frac{1}{5}, t > \frac{5}{2}$

D. $\frac{1}{5} < t < \frac{5}{2}$ E. $t > \frac{1}{5}$

---

59. A particle moving along the *x*-axis such that at any time $t \geq 0$ its velocity is given by $v(t) = \frac{t^3}{\ln\sqrt{t}}$. What is the acceleration of the particle at $t = e$?

A. $6e^2 - 2e$ B. $6e^3$ C. $2e^3$ D. $4e^2$ E. $6e^2 - 4e^{5/2}$

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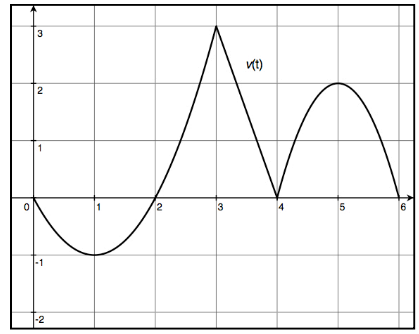
60. (**Calc**) A particle moves along a straight line with velocity given by $v(t) = t - 2.5^{\cos 2t}$. What is the acceleration of the particle at $t = 2$?

A. 0.168 B. 0.238 C. 0.451 D. 0.584 E. 1.450

61. A squirrel climbs a telephone pole and starts to walk along the telephone wire. Its velocity v of the squirrel at time t, $0 \le t \le 6$ is given by the function whose graph is to the right. At what value of t does the squirrel change direction?



A. 1 and 5 only B. 2 only C. 2 and 4 only

D. 1, 3, and 5 only E. 1, 3, 4 and 5 only

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62. A particle moving along a straight line that at any time $t \ge 0$ its velocity is given by $v(t) = \dfrac{\sin t}{\cos 2t}$. For which values of t is the particle speeding up?

I. $t = \dfrac{\pi}{6}$ II. $t = \dfrac{\pi}{3}$ III. $t = \dfrac{2\pi}{3}$

A. I only B. II only C. III only D. I and II only E. I and III only

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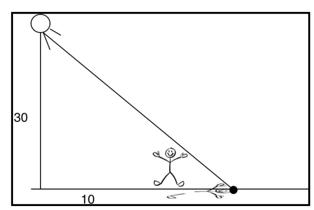
63. Frank, who is 6 feet tall, is performing along a bare stage as shown in the figure to the right. There is a single spotlight 30 feet above the left wing of the stage and Frank is walking across the stage from left to right at 3 feet/sec. When Frank is 10 feet from the left edge of the stage, how fast is the tip of his shadow moving across the stage?



A. 15 ft/sec B. $\dfrac{3}{4}$ ft/sec C. $\dfrac{15}{4}$ ft/sec D. $\dfrac{3}{5}$ ft/sec E. $\dfrac{18}{5}$ ft/sec