L. Function Analysis

What you are finding: You have a function $f(x)$. You want to find intervals where $f(x)$ is increasing and decreasing, concave up and concave down. You also want to find values of x where there is a relative minimum, a relative maximum, and points of inflection.
How to find them: Find critical values – values $x = c$ where $f'(c) = 0$ or $f'(c)$ is undefined.

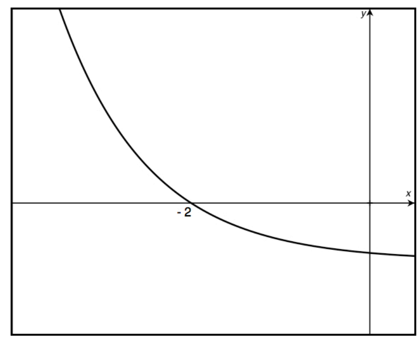
$f(x)$ is increasing for values of k such that $f'(k) > 0$
$f(x)$ is decreasing for values of k such that $f'(k) < 0$.

If $f(x)$ switches from increasing to decreasing at c, there is a relative maximum at $(c, f(c))$. If $f(x)$ switches from decreasing to increasing at c, there is a relative minimum at $(c, f(c))$. This is commonly called the first derivative test.

$f(x)$ is concave up for values of k such that $f''(k) > 0$.
$f(x)$ is concave down for values of k such that $f''(k) < 0$.

If $f(x)$ switches concavity at c, there is a point of inflection at $(c, f(c))$. If you want to find the actual maximum or minimum value or find the value of the function at specific points, you need to use accumulated area and the FTC.

64. The graph of a twice-differentiable function f is shown in the figure to the right. Which of the following is true?



A. $f(-2) < f'(-2) < f''(-2)$
B. $f(-2) < f''(-2) < f'(-2)$
C. $f''(-2) < f'(-2) < f(-2)$
D. $f'(-2) < f(-2) < f''(-2)$
E. $f''(-2) < f(-2) < f'(-2)$

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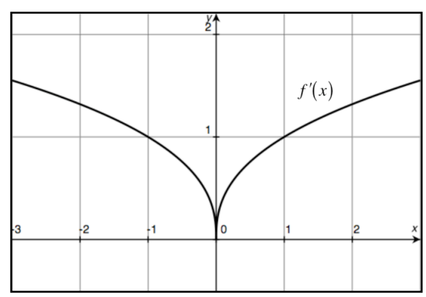
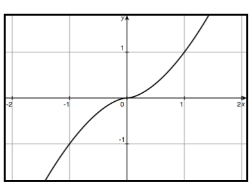
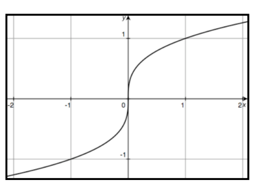
65. If the graph of $f'(x)$ is shown to the right, which of the following could be the graph of $y = f(x)$?

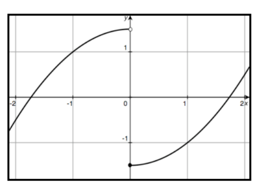




I.



II.



III.

A. I only B. II only C. III only D. I and III only E. II and III only

66. **(Calc)** The first derivative of a function f is given by $f'(x) = (x-2)e^{\sin 2x}$. How many points of inflection does the graph of f have on the interval $0 < x < 2\pi$?

A. Two B. Three C. Four D. Five E. Six

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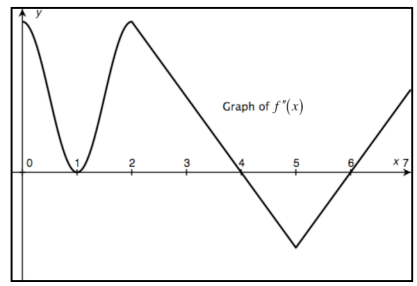
67. The graph of f'', the second derivative of f, is shown to the right. On which of the following intervals is $f'(x)$ decreasing?



A. [0, 1]
B. [0, 1] and [2, 4]
C. [0, 1] and [2, 5]
D. [4, 5]
E. [4, 6]

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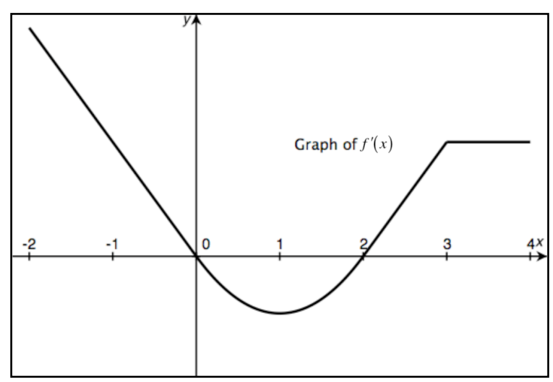
68. The graph of $f'(x)$, the derivative of f, is shown to the right. Which of the following statements is **not** true?



A. f is increasing on $2 \le x \le 3$.
B. f has a local minimum at $x = 1$.
C. f has a local maximum at $x = 0$.
D. f is differentiable at $x = 3$.
E. f is concave down on $-2 \le x \le 1$.

69. If $f'(x)=(2x-1)^2(3x+1)^3(x-1)$, then f has which of the following relative extrema?

I. A relative maximum at $x=\frac{-1}{3}$

II. A relative minimum at $x=\frac{1}{2}$

III. A relative minimum at $x=1$

A. I only B. III only C. I and II only D. I and III only E. I, II and III

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70. **(Calc)** Let $f'(x)=(x-1)^2\cos(x^2+1)$. If c represents the largest value on the x-axis on [0, 2] where there is a point of relative extrema of $f(x)$, and let d represents the smallest value on the x-axis on [0, 2] where there is a point of relative extrema of $f(x)$, find the value of $c-d$.

A. 0.244 B. 0.927 C. 1.171
D. -0.689 E. there are no relative extrema

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71. Let g be a strictly decreasing function such that $g(x)<0$ for all real numbers x. If $f(x)=(x-1)^2 g(x)$, which of the following is true?

A. f has a relative minimum at $x = 1$
B. f has a relative maximum at $x = 1$
C. f will be a strictly decreasing function
D. f will be a strictly increasing function
E. It cannot be determined if f has a relative extrema

72. If $f'(x) = \dfrac{x^2 - 5x - 5}{e^{-x}}$, for what values of x is $f(x)$ concave down?

A. All values of x B. No values of x C. $-2 < x < 5$

D. $x > 5$ or $x < -2$ E. $\dfrac{1-\sqrt{53}}{2} < x < \dfrac{1+\sqrt{53}}{2}$

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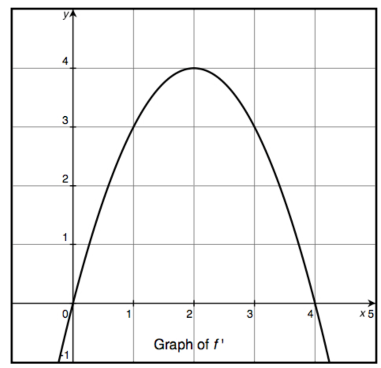
73. The function g is defined by $g(x) = x^2 - f(x)$ where the graph of $f'(x)$ is pictured to the right. Find and classify all points of relative extrema of $g(x)$.



A. Relative maximum at $x = 2$ only
B. Relative minimum at $x = 2$ only
C. Relative maximum at $x = 2$ and relative minimum at $x = 0$
D. Relative maximum at $x = 0$ and relative minimum at $x = 2$
E. Relative minimum at $x = 0$ and relative maximum at $x = 4$

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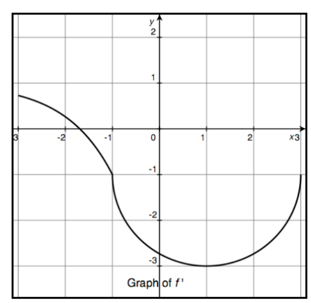
74. The derivative of a function f is defined by $f'(x) = \begin{cases} g(x), & -1 \le x \le 1 \\ 1 - 2e^{x+1}, & -3 \le x < -1 \end{cases}$.

The graph of the continuous function f' is shown to the right with the graph of g being a semicircle. Which of the following statements are true?



I. f has a relative maximum at $x = -1 - \ln 2$
II. f has an inflection point at $x = 1$
III. f is increasing on (1,3)

A. I only B. II only C. I and II only D. II and III only E. I, II and III

M. Second Derivative Test

What you are finding: Students normally find points of relative maximum and relative minimum of $f(x)$ by using the first derivative test, finding points where a function switches from increasing to decreasing or vice versa. But students may be forced to use the 2nd derivative test, as seen below. However, this type of problem is rare on the multiple choice section. **How to use it:** Find critical values c where $f'(c)=0$. a) If $f''(c)>0$, f is concave up at c and there is a relative minimum at $x=c$. b) If $f''(c)<0$, f is concave down at c and there is a relative maximum at $x=c$. c) If $f''(c)=0$, the 2nd derivative test is inconclusive. $\left(\text{ex.} f(x)=x^4 \text{ at } x=0\right)$

75. If $f(x)=k\sin x+\ln x$ where k is a constant has a critical point at $x=\pi$, determine which statement below is true.

A. f has a relative minimum at $x=\pi$
B. f has a relative maximum at $x=\pi$
C. f has a point of inflection at $x=\pi$
D. f has no concavity at $x=\pi$
E. None of the above

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76. Consider the differential equation $\dfrac{dy}{dx}=x^2-2y+3$. If $y=f(x)$ is the solution to the differential equation, at what point does f have a relative minimum?

A. (0, 0)
B. (-1, 2)
C. (4, 9)
D. $\left(0,\dfrac{3}{2}\right)$
E. (5, 14)

N. Absolute Extrema

What you are finding: the highest value or lowest value of a function, typically in an interval. Another way of asking the question is to ask for the range of the function. Although these types of problems use differential calculus, students can be asked to utilize the fact that $f(x) = f(0) + \int_0^x f'(t)\,dt$ to find an extreme value of f on an interval when either given an expression for $f'(x)$ or a graphical representation for it.

How to find it: a) find critical points (x-values where the derivative = 0 or is not defined).
b) use 1st derivative test to determine x-values when the function is increasing/decreasing.
c) evaluate function at critical points and at the endpoints. This may involve using the FTC to find accumulated change.

77. The absolute maximum value of $y = \left|x^2 - x - 12\right|$ on $-4 \le x \le 4$ is

A. 0.5 B. 4 C. 8 D. 11.75 E. 12.25

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78. The difference in maximum acceleration and minimum acceleration attained on the interval $0 \le t \le 3$ by the particle whose velocity is given by $v(t) = 2t^3 - 12t^2 + 18t - 1$ is

A. 4 B. 6 C. 18 D. 21 E. 24

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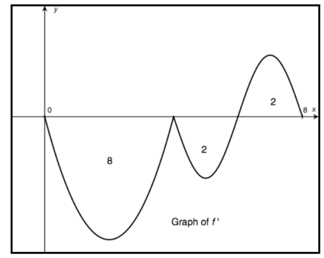
79. **(Calc)** The rate at which the gasoline is changing in the tank of a hybrid car is modeled by $f(t) = \sqrt{t} + .5\sin t - 2.5$ gallons per hour, t hours after a 6-hour trip starts. At what time during the 6-hour trip was the gasoline in the tank going down most rapidly?

A. 0 B. 2.292 C. 3.228 D. 4.203 E. 6

80. The graph of f', the derivative of f, is shown to the right for $0 \le x \le 8$. The areas of the regions between the graph of f' and the axis are 8, 2 and 2 respectively. If $f(0) = 10$, what is the minimum value of f on the interval $0 \le x \le 8$?



A. -12 B. -10 C. -8
D. 0 E. 10

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81. **(Calc)** Let f be the function with the first derivative defined as $f'(x) = \sin(x^3 - 4x + 2)$ for $0 \le x \le 2$. Let c be the value of x where f attains its minimum value in the interval $0 \le x \le 2$ and let d be the value of x where f attains its maximum value in the interval $0 \le x \le 2$. Find $|c - d|$.

A. 0.539 B. 1.136 C. 1.461 D. 1.675 E. 2

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82. **(Calc)** Cellophane-wrapped Tastycakes come off the assembly line at the rate of $C(t) = 5 + 4\sin\left(\frac{\pi t}{6}\right)$. They are boxed at the rate of $B(t) = \frac{20t}{1+t^2}$. $C(t)$ and $B(t)$ are measured in hundreds of cakes per hour, and t is measured in hours for $0 \le t \le 5$. There are 500 cakes waiting to be boxed at the start of the 5-hour shift. How long (in hours) does it take the number of cakes waiting to be boxed to go from a minimum to a maximum during the 5-hour shift?

A. 1.560 B. 1.869 C. 3.131 D. 4.691 E. 5

O. Computation of Riemann Sums

What you are finding: Riemann sums are approximations for definite integrals, which we know represent areas under curves. There are numerous real-life models for areas under curves so this is an important concept. Typically these types of problems show up when we are given data points as opposed to algebraic functions.

How to find them: Given data points:

x	x_0	x_1	x_2	...	x_{n-2}	x_{n-1}	x_n
$f(x)$	$f(x_0)$	$f(x_1)$	$f(x_2)$	...	$f(x_{n-2})$	$f(x_{n-1})$	$f(x_n)$

Assuming equally spaced x-values: $x_{i+1}-x_i=b$

Left Riemann Sums: $S=b\left[f(x_0)+f(x_1)+f(x_2)+...+f(x_{n-1})\right]$

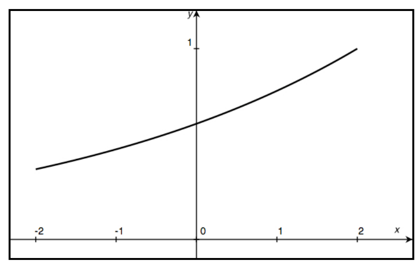
Right Riemann Sums: $S=b\left[f(x_1)+f(x_2)+f(x_3)+...+f(x_n)\right]$

Trapezoids: $S=\dfrac{b}{2}\left[f(x_0)+2f(x_1)+2f(x_2)+...+2f(x_{n-2})+2f(x_{n-1})+f(x_n)\right]$

If bases are not the same (typical in AP questions), you have to compute the area of each trapezoid: $\dfrac{1}{2}(x_{i+1}-x_i)\left[f(x_i)+f(x_{i+1})\right]$

Midpoints: This is commonly misunderstood. For example, you cannot draw a rectangle halfway between $x=2$ and $x=3$ because you may not know $f(2.5)$. You can't make up data. So in the table above the first midpoint rectangle would be drawn halfway between x_0 and x_2 which is x_1. So assuming that the x-values are equally spaced, the midpoint sum is $S=2b\left[f(x_1)+f(x_3)+f(x_5)+...\right]$.

83. The graph of the function f is shown to the right for $-2 \le x \le 2$. Four calculations are made:



LS – Left Riemann sum approximation of $\int_{-2}^{2} f(x)\,dx$ with 4 subintervals of equal length

RS – Right Riemann sum approximation of $\int_{-2}^{2} f(x)\,dx$ with 4 subintervals of equal length

TS – Trapezoidal sum approximation of $\int_{-2}^{2} f(x)\,dx$ with 4 subintervals of equal length

DI – $\int_{-2}^{2} f(x)\,dx$

Arrange the results of the calculations from highest to lowest.

A. DI – RS – TS - LS
B. RS – TS – DI - LS
C. TS – DI – LS - RS
D. LS – DI – TS - RS
E. RS – DI – TS - LS

84. The function f is continuous on the closed interval [0, 8] and has the values given in the table below. The trapezoidal approximation for $\int_0^8 f(x)\,dx$ found with 3 subintervals is $20k$. What is the value of k?

x	0	3	5	8
$f(x)$	5	k^2	7	10

A. 4 B. ±4 C. 8 D. -8 E. No values of k

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85. The expression $\frac{1}{75}\left(\ln\frac{76}{75}+\ln\frac{77}{75}+\ln\frac{78}{75}+...+\ln 2\right)$ is a Riemann sum approximation for

A. $\int_1^2 \ln\left(\frac{x}{75}\right)dx$ B. $\int_{76}^{150} \ln\left(\frac{x}{75}\right)dx$ C. $\frac{1}{75}\int_{76}^{150} \ln x\,dx$ D. $\int_1^2 \ln x\,dx$ E. $\frac{1}{75}\int_1^2 \ln x\,dx$

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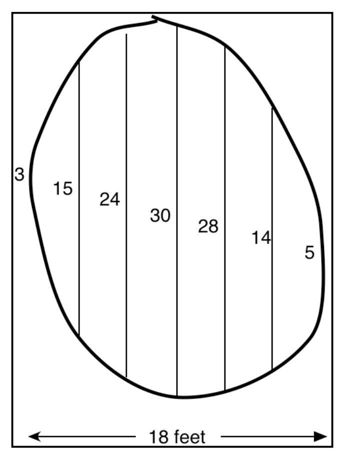
86. An oil slick forms in a lake. The oil is 4 feet deep. The slick is 18 feet wide and is divided into 6 sections of equal width having measurements as shown in the figure to the right. Tim uses a trapezoidal sum with six trapezoids to approximate the volume while Doug uses a midpoint sum using three equally spaced rectangles. What is the difference between their approximations of the volume of the slick in cubic feet?



A. 9
B. 36
C. 12
D. 3
E. 0

P. Integration Techniques

What you are finding: Integration is anti-differentiation. If $\frac{d}{dx} f(x)\, dx = g(x)$ then $\int g(x)\, dx = f(x) + C$. While a derivative can be taken for any expression, not all expressions can be integrated. Just as differentiation of complicated expressions involve u-substitution, substitution can be used to integrate some expressions. It is important to add on the constant of integration when performing anti-differentiation.

How to find it: These are usually straightforward. Students need to know these integration formulas.

$\int u^n du = \frac{u^{n+1}}{n+1} + C$	$\int \frac{du}{u} = \ln\lvert u\rvert + C$	$\int e^u = e^u + C$	$\int a^u = \frac{1}{\ln a} a^u + C$
$\int \sin u\, du = -\cos u + C$	$\int \cos u\, du = \sin u + C$	$\int \sec^2 u\, du = \tan u + C$	
$\int \csc u\, du = -\cot u + C$	$\int \sec u \tan u = \sec u + C$	$\int \csc u \cot u\, du = -\csc u + C$	
$\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1} \frac{u}{a} + C$	$\int \frac{du}{a^2 + u^2} = \frac{1}{a} \tan^{-1} \frac{u}{a} + C$	$\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \sec^{-1} \frac{u}{a} + C$	

87. $\int \left(x^4 - x\right)^2 dx =$

A. $\frac{x^9}{9} + \frac{x^3}{3} + C$ B. $\frac{x^9}{9} - \frac{x^6}{6} + \frac{x^3}{3} + C$ C. $\frac{x^9}{9} - \frac{x^6}{3} + \frac{x^3}{3} + C$

D. $\frac{\left(x^4 - x^2\right)^3}{3} + C$ E. $\frac{\left(x^4 - x^2\right)^3}{3\left(4x^3 - 2x\right)} + C$

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88. $\int \frac{e^{\sqrt{x+2}}}{\sqrt{8x+16}}\, dx =$

A. $2e^{\sqrt{x+2}} + C$ B. $\frac{e^{\sqrt{x+2}}}{2} + C$ C. $\frac{e^{\sqrt{x+2}}}{8} + C$

D. $\sqrt{2}e^{\sqrt{x+2}} + C$ E. $\frac{\sqrt{2}e^{\sqrt{x+2}}}{2} + C$

89. $\int \frac{x+2}{x^2+4}\, dx =$

A. $\frac{\ln\left(x^2+4\right)+\tan^{-1}(x)}{2}+C$

B. $\tan^{-1}\left(\frac{x}{2}\right)+C$

C. $\ln(x+2)+C$

D. $\frac{1}{2}\ln\left(x^2+4\right)+\tan^{-1}\left(\frac{x}{2}\right)+C$

E. $\frac{1}{2}\left[\ln\left(x^2+4\right)+\tan^{-1}\left(\frac{x}{2}\right)\right]+C$

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90. $\int \frac{\sin 2x}{1-\sin^2 2x}\, dx =$

A. $2\ln(\cos 2x)+C$

B. $\frac{1}{2}\ln(\cos 2x)+C$

C. $\frac{1}{2}\sec 2x \tan 2x + C$

D. $\frac{1}{2}\sec 2x + C$

E. $2\tan 2x + C$

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91. **(Calc)** Let $F(x)$ be an antiderivative of $\frac{(\ln x - 3)^2}{x}$. If $F(1)=2$, then $F(2)=$

A. -11.092 B. -8.769 C. 2.092 D. 6.908 E. 16.724

Q. Fundamental Theorem of Calculus (FTC) / Accumulation Function

What you are finding: Students should certainly know that the FTC says that $\int_a^b f(x)\,dx = F(b) - F(a)$ where $F(x)$ is an antiderivative of $f(x)$. This leads to the fact that $F(a) + \int_a^b f(x)\,dx$. Students should be prepared to use substitution methods to integrate and be able to change the limits of integration using this substitution. Students should also know that $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$.

The accumulation function looks like this: $F(x) = \int_0^x f(t)\,dt$. It represents the accumulated area under the curve *f* starting at zero (or some value) and going out to the value of *x*. The variable *t* is a dummy variable. It is important to believe that this is a function of x.

92. $\int_0^{\pi} e^{\cos x} \sin x\,dx =$

A. $\frac{1}{e}$ B. $e - \frac{1}{e}$ C. $e - 1$ D. $1 - \frac{1}{e}$ E. e

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93. $\int_1^3 \sqrt{20 - 4x}\,dx$ is equivalent to

A. $\frac{-1}{4}\int_{-1/5}^{1/5} \sqrt{u}\,du$ B. $\frac{-1}{4}\int_1^3 \sqrt{u}\,du$ C. $\frac{1}{4}\int_8^{16} \sqrt{u}\,du$ D. $4\int_8^{16} \sqrt{u}\,du$ E. $\int_8^{16} \sqrt{u}\,du$

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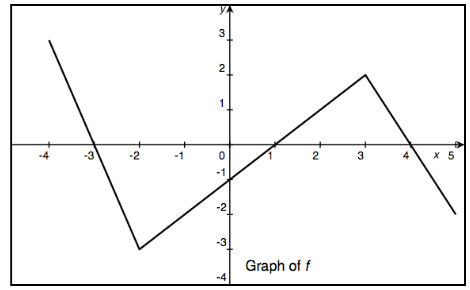
94. $\int_1^x \frac{\ln t}{t}\,dt =$

A. $\frac{x^2 - 1}{2}$ B. $\frac{x^2}{2}$ C. $\frac{(\ln x)^2}{2}$ D. $\frac{(\ln x)^2 - 1}{2}$ E. $\ln x$

95. The graph of the piecewise linear function f is shown in the figure to right. If $g(x)=\int_1^x f(t)\,dt$, which of the following is the greatest?



A. $g(-4)$ B. $g(-3)$ C. $g(1)$
D. $g(4)$ E. $g(5)$

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96. **(Calc)** Let $F(x)$ be an antiderivative of $\sqrt{x^3+x+1}$. If $F(1)=-2.125$, then $F(4)=$

A. -15.879 B. -11.629 C. 7.274 D. 15.879 E. 11.629

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97. If f is a continuous function and $F'(x)=f(x)$ for all real numbers x, then $\int_{-2}^{2} f(1-3x)\,dx=$

A. $3F(-2)-3F(2)$ B. $\frac{1}{3}F(-2)-\frac{1}{3}3F(2)$ C. $\frac{1}{3}F(2)-\frac{1}{3}3F(-2)$
D. $3F(-5)-3F(7)$ E. $\frac{1}{3}F(7)-\frac{1}{3}F(-5)$

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98. If f is continuous for all real numbers x and $\int_1^4 f(x)\,dx=10$, then $\int_3^6 \left[f(x-2)+2x\right]dx=$

A. 37 B. 39 C. 35 D. 57 E. 25

R. Derivative of Accumulation Function (2nd FTC)

What you are finding: You are looking at problems in the form of $\frac{d}{dx}\int_a^x f(t)\,dt$. This is asking for the rate of change with respect to x of the accumulation function starting at some constant (which is irrelevant) and ending at that variable x. It is important to understand that this expression is a function of x, not the variable t. In fact, the variable t in this expression could be any variable (except x).

How to find it: You are using the 2nd Fundamental Theorem of Calculus that says: $\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$.

Occasionally you may have to use the chain rule that says $\frac{d}{dx}\int_a^{g(x)} f(t)\,dt = f(g(x))\cdot g'(x)$.

99. Let f be the function given by $f(x)=\int_x^{\pi/2} t\sin 2t\,dt$ for $-\pi \le x \le \pi$. In what interval(s) is $f(x)$ decreasing?

A. $(0,\pi)$ B. $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ C. $\left(-\pi,\frac{-\pi}{2}\right)\cup\left(\frac{\pi}{2},\pi\right)$

D. $(-\pi,0)$ E. $(-\pi,\pi)$

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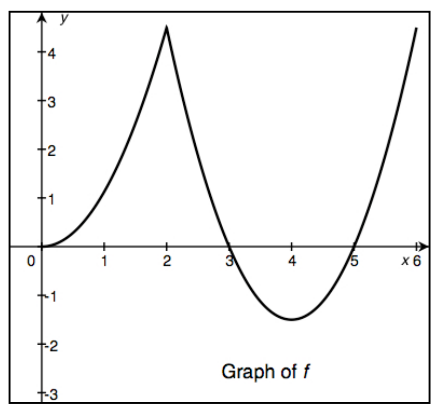
100. The graph of the function f shown to the right has a horizontal tangent at $x = 4$. Let g be the continuous function defined by $g(x)=\int_0^x f(t)\,dt$. For what value(s) of x does the graph of g have a point of inflection?

A. 3 and 5 B. 0, 3, and 5 C. 2 only

D. 2 and 4 E. 4 only



101. $\dfrac{d}{dx}\displaystyle\int_1^{x^3} \tan\left(t^4 - 1\right)\, dt =$

A. $\sec^2\left(x^{12} - 1\right)$ B. $\tan\left(x^4 - 1\right)$ C. $\tan\left(x^{12} - 1\right)$
D. $3x^2 \tan\left(x^{12} - 1\right)$ E. $12x^{11} \tan\left(x^{12} - 1\right)$

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102. **(Calc)** Let f be the function given by $f(x) = \displaystyle\int_0^x \cos\left(t^2 + t\right)\, dt$ for $-2 \le x \le 2$. Approximately, for what percentage of values of x for $-2 \le x \le 2$ is $f(x)$ decreasing?

A. 30% B. 26% C. 44% D. 50% E. 59%

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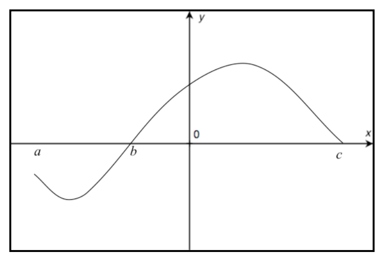
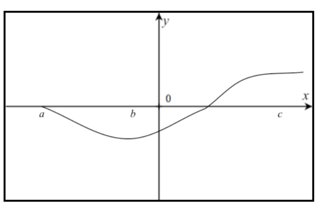
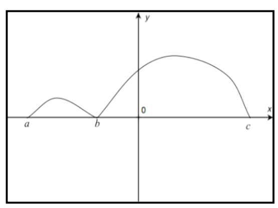
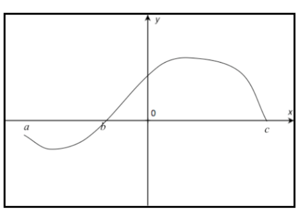
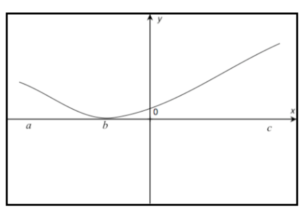
103. Let $f(x) = \displaystyle\int_a^x g(t)\, dt$ where g has the graph shown to the right. Which of the following could be the graph of f?

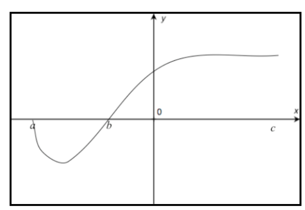


A. 

B. 

C. 

D. 

E. 

S. Interpretation of a Derivative as a Rate of Change

What you are finding: As mentioned in section I (Related Rates), a quantity that is given as a rate of change needs to be interpreted as a derivative of some function. Typical problems ask for the value of the function at a given time. These problems can be handled several ways:

a) solving a Differential Equation with initial condition (although DEQ's may not have even been formally mentioned yet)

b) Integral of the rate of change to give accumulated change. This uses the fact that:

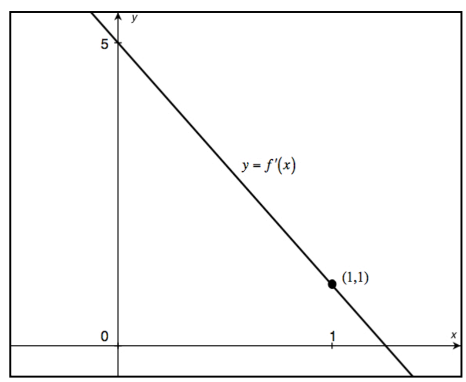
$$\int_a^b R'(t)\,dt = R(b) - R(a) \text{ or } R(b) = R(a) + \int_a^b R'(t)\,dt$$

104. The table below gives values of a function f and its derivative at selected values of x. If f' is continuous on the interval [-4, 4], what is the value of $\int_{-2}^{2} f'(2x)\,dx$?

x	-4	-2	-1	1	2	4
$f(x)$	8	-6	-4	-2	-4	4
$f'(x)$	-6	-4	-2	0	2	4

A. -2 B. -8 C. 5 D. 20 E. 1

105. The graph of $y = f'(x)$, the derivative of f, is shown in the figure to the right. If $f(0) = -1$, then $f(1) =$



A. 1 B. 2 C. 3

D. 4 E. 5

106. **(Calc)** The rate of change of people waiting in line to buy tickets to a concert is given by $w(t) = 100\left(t^3 - 4t^2 - t + 7\right)$ for $0 \le t \le 4$. 800 people are already waiting in line when the box office opens at $t = 0$. Which of the following expressions give the change in people waiting in line when the line is getting shorter?

A. $\int_{1.480}^{3.773} w'(t)\,dt$

B. $\int_{1.480}^{3.773} w(t)\,dt$

C. $800 - \int_{1.480}^{3.773} w'(t)\,dt$

D. $\int_{0}^{2.786} w'(t)\,dt$

E. $\int_{0}^{2.786} w(t)\,dt$

---

107. **(Calc)** A cup of coffee is heated to boiling $(212°\text{F})$ and taken out of a microwave and placed in a $72°\text{F}$ room at time $t = 0$ minutes. The coffee cools at the rate of $16e^{-0.112t}$ degrees Fahrenheit per minute. To the nearest degree, what is the temperature of the coffee at time $t = 5$ minutes?

A. 105°F B. 133°F C. 166°F D. 151°F E. 203°F

---

108. **(Calc)** The Cheesesteak Factory at a Philadelphia stadium has 25 steak sandwiches ready to sell when it opens for business. It cooks cheesesteaks at the rate of 4 steaks per minute and sells cheesesteaks at the rate of $1 + 6\sin\left(\frac{2\pi t}{41}\right)$ steaks per minute. For $0 \le t \le 20$, at what time is the number of steaks ready to sell at a minimum (nearest tenth of a minute)?

A. 0 B. 3.4 C. 10.3 D. 17.1 E. 20

T. Average Value of a Function

What you are finding: You are given a continuous function $f(x)$, either an algebraic formula or a graph, as well as an interval $[a,b]$. You wish to find the average value of the function on that interval.

How to find it: $f_{avg} = \dfrac{\int_a^b f(x)\,dx}{b-a}$. The units will be whatever the function *f* is measured in.

Again, be careful. The average rate of change of a function *F* on $[a,b]$ is not the same as the average value of the function *F* on $[a,b]$.

Average value of the function: $\dfrac{\int_a^b f(x)\,dx}{b-a}$

Average rate of change of the function (average value of the rate of change): $\dfrac{\int_a^b F'(x)\,dx}{b-a} = \dfrac{F(b)-F(a)}{b-a}$

109. The average value of $\sin^2 x \cos x$ on the interval $\left[\dfrac{\pi}{2}, \dfrac{3\pi}{2}\right]$ is

A. $\dfrac{-2}{3\pi}$ B. $\dfrac{2}{3\pi}$ C. 0 D. -1 E. 1

---

110. The velocity of a particle moving along the *x*-axis is given by the function $v(t) = te^{t^2}$. What is the average velocity of the particle from time $t = 1$ to $t = 3$?

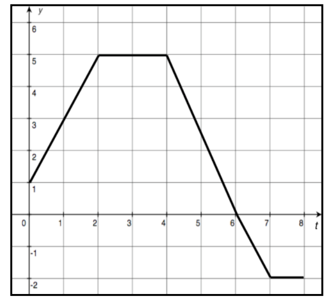
A. $\dfrac{19e^9 - 3e}{2}$ B. $4e^9$ C. $\dfrac{3e^9 - e}{2}$ D. $\dfrac{e^9 - e}{4}$ E. $e^9 - e$

---

111. **(Calc)** What is the average value of $y = \dfrac{\sin(2x+3)}{x^2+2x+3}$ on the interval [-2, 3] ?

A. -0.057 B. 0.051 C. 0.061 D. 0.256 E. 0.303

112. Newton the cat begins to walk along a ledge at time $t = 0$. His velocity at time t, $0 \le t \le 8$, is given by the function whose graph is given in the figure to the right. What is Newton's average speed from $t = 0$ to $t = 8$?



A. 0 B. 2 C. 3

D. $\frac{9}{4}$ E. 5

---

113. **(Calc)** Barstow, California has the daily summer temperature modeled by $T(t) = 85 + 27\sin\left[\frac{\pi(x-9)}{12}\right]$ where t is measured in hours and $t = 0$ corresponds to 12:00 midnight. What is the average temperature in Barstow during the period from 1:30 PM to 6 PM?

A. 60° B. 89° C. 104° D. 110° E. 115°

---

114. Let $f(x) = 3x^2 - 6x$. For how many positive values of b does the average value of $f(x)$ on the interval [0, b] equal the average rate of change of $f(x)$ on the interval [0, b] ?

A. 4 B. 3 C. 2 D. 1 E. none

What you are finding: In section J, we looked at straight-line motion by the derivative process. You were typically given a position function $x(t)$ and took its derivative to find the velocity function $v(t)$ and took the velocity's derivative to get the acceleration function $a(t)$. Questions like finding maximum velocity or minimum acceleration could then be answered.

Using integrals, we can work backwards. Typical questions involve knowing the velocity function, the position of the particle at the start of the problem, and integrating to find the position function.

How to find it: $x(t)=\int v(t)\,dt+C$ and $v(t)=\int a(t)\,dt+C$. Using these equations is essentially solving DEQ's with an initial condition, usually the position at time $t = 0$ or velocity at $t = 0$. Two other concepts that come into play are displacement and distance over some time interval $[t_1,t_2]$.

<u>Displacement:</u> the difference in position over $[t_1,t_2]$: $x(t_2)-x(t_1)=\int_{t_1}^{t_2} v(t)\,dt$. Displacement can be positive, negative or zero.

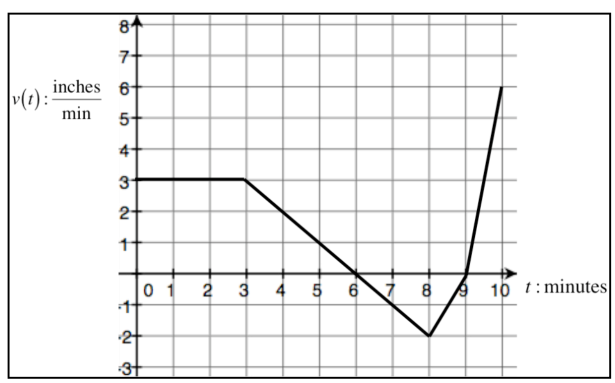
<u>Distance</u>: how far the particle traveled over $[t_1,t_2]$: $\int_{t_1}^{t_2} |v(t)|\,dt$. Distance is always positive.

115. A particle moves along the x-axis so that at any time $t \geq 0$, its velocity is given by $v(t)=5-2t-3t^2$. If the particle is at position $x=-3$ at time $t = 2$, where was the particle at $t = 1$?

A. -5 B. 2 C. 3 D. 4 E. 11

---

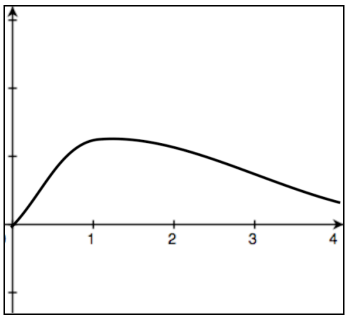
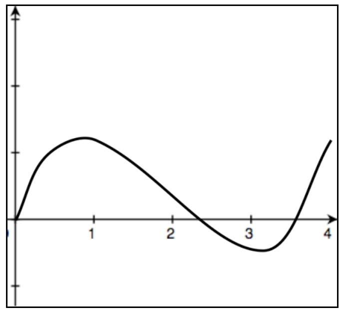
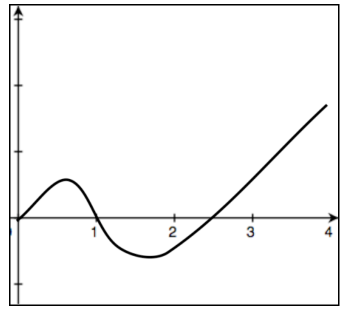
116. A spider is walking up a vertical wall. At $t = 0$, the spider is 18 inches off the floor. The velocity v of the spider, measured in inches per minute at time t minutes, $0 \leq t \leq 10$ is given by the function whose graph is shown to the right. How many inches off the floor is the spider at $t = 10$?

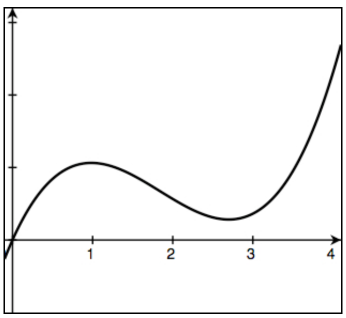
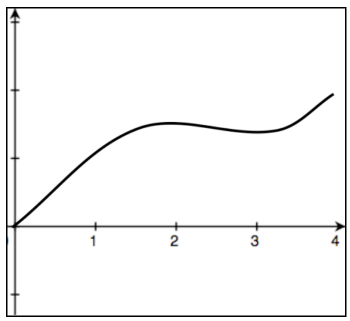


A. 13.5 B. 19.5 C. 24
D. 31.5 E. 37.5

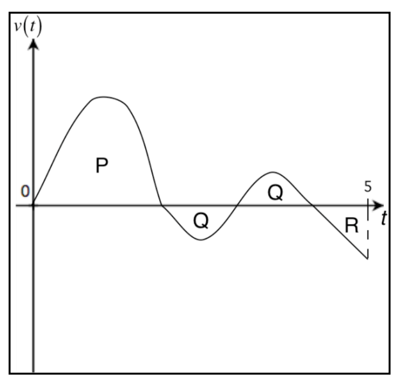
117. The table below gives selected values of $v(t)$, of a particle moving along the x-axis. At time $t = 0$, the particle is at the origin. Which of the following could be the graph of the position, $x(t)$, of the particle for $0 \le t \le 4$?

t	0	1	2	3	4
$v(t)$	3	0	-1	1	3

A. 
B. 
C. 

D. 
E. 

118. A particle moves along the y-axis with velocity $v(t)$ for $0 \le t \le 5$. The graph of $v(t)$ is shown to the right with the positive areas indicated by P, Q, and R. What is the difference between the distance that the particle travels and its displacement for $0 \le t \le 5$?



A. P - R B. 2(P + Q + R) C. 2Q
D. 2R E. 2(Q + R)

119. **(Calc)** A particle moves along the x-axis so that its velocity $v(t) = 12te^{-2t} - t + 1$. At $t = 0$, the particle is at position $x = 0.5$. What is the total distance that the particle traveled from $t = 0$ to $t = 3$?

A. 1.448 B. 1.948 C. 2.911 D. 4.181 E. 4.681

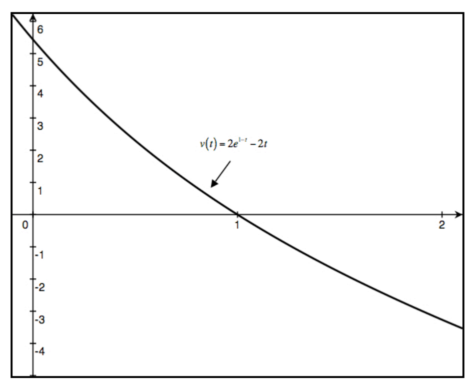
---

120. **(Calc)** A particle moves along the x-axis so that at any time $t > 0$, its acceleration is given by $a(t) = \cos(1 - 2^t)$. If the velocity of the particle is -2 at time $t = 0$, then the speed of the particle at $t = 2$ is

A. 0.613 B. 0.669 C. 1.331 D. 1.387 E. 1.796

---

121. A particle moves along the x-axis so that its velocity $v(t) = 2e^{1-t} - 2t$, as shown in the figure to the right. Find the total distance that the particle traveled from $t = 0$ to $t = 2$.



A. $2e + \frac{2}{e} - 2$ B. $2e - \frac{2}{e} - 4$

C. $2e - \frac{2}{e} + 4$ D. $2e - 2$

E. $2e + \frac{2}{e} + 2$

V. Area/Volume Problems

What you are finding: Typically, these are problems with which students feel more comfortable because they are told exactly what to do or to find. Area and volume are lumped together because, almost always, they are both tested within the confines of a single A.P. free response question. Usually, but not always, they are on the calculator section of the free-response section.

Area problems usually involve finding the area of a region under a curve or the area between two curves between two values of x. Volume problems usually involve finding the volume of a solid when rotating a curve about a line.

How to find it: Area: Given two curves $f(x)$ and $g(x)$ with $f(x) \ge g(x)$ on an interval [a, b], the area between f and g on [a, b] is given by $A = \int_{x=a}^{x=b} \left[f(x) - g(x)\right] dx$. While integration is usually done with respect to the x-axis, these problems sometimes show up in terms of y: $A = \int_{y=c}^{y=d} \left[m(y) - n(y)\right] dy$

Volume: Disks and Washers: The method I recommend is to establish the outside Radius R, the distance from the line of rotation to the outside curve, and, if it exists, the inside radius r, the distance from the line of rotation to the inside curve. The formula when rotating these curves about a line on an interval is given by:

$$V = \pi \int_{x=a}^{x=b} \left(\left[R(x)\right]^2 - \left[r(x)\right]^2\right) dx \quad \text{or} \quad V = \pi \int_{y=c}^{y=d} \left(\left[R(y)\right]^2 - \left[r(y)\right]^2\right) dy$$

A favorite type of problem is creating a solid with the region R being the base of the solid. Cross sections perpendicular to an axis are typically squares, equilateral triangles, right triangles, or semi-circles. Rather than give formulas for this, it is suggested that you draw the figure, establish its area in terms of x or y, and integrate that expression on the given interval.

122. What is the area of the region enclosed by the graphs of $y = x - 4x^2$ and $y = -7x$?

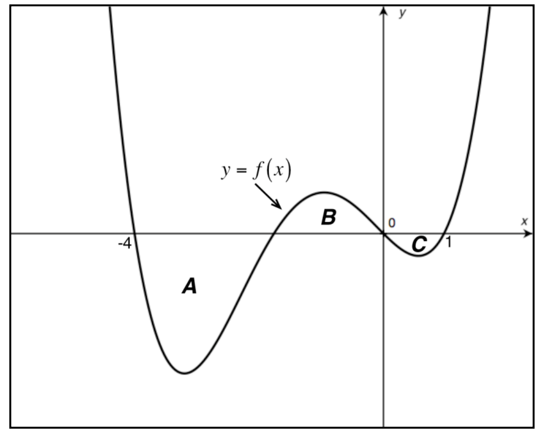
A. $\frac{4}{3}$ B. $\frac{16}{3}$ C. 8 D. $\frac{68}{3}$ E. $\frac{80}{3}$

---

123. **(Calc)** What is the area of the region in the first quadrant enclosed by the graph of $y = 2\cos x, y = x$, and the x-axis?

A. 0.816 B. 1.184 C. 1.529 D. 1.794 E. 1.999

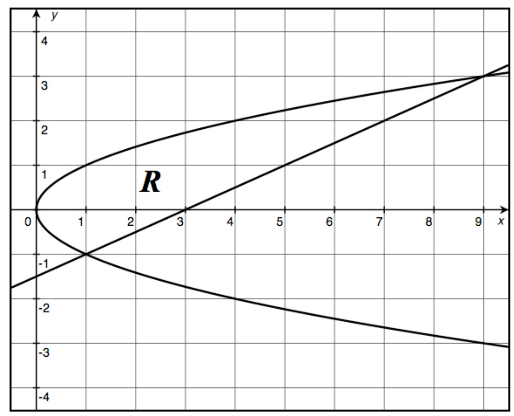
124. The three regions A, B, and C in the figure to the right are bounded by the graph of the function f and the x-axis. If the areas of A, B, and C are 5, 2, and 1 respectively, what is the value of $\int_{-4}^{1}\left[f(x)+2\right]dx$?



A. 28
B. 10
C. 6
D. -4
E. -12

---

125. Region R, enclosed by the graphs of $y=\pm\sqrt{x}$ and $y=\dfrac{x-3}{2}$ is shown in the figure to the right. Which of the following calculations would accurately compute the area of region R?



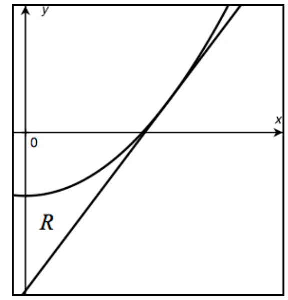
I. $\displaystyle\int_{0}^{9}\left(\sqrt{x}-\frac{x-3}{2}\right)dx$

II. $\displaystyle\int_{0}^{1}2\sqrt{x}\,dx+\int_{1}^{9}\left(\sqrt{x}-\frac{x-3}{2}\right)dx$

III. $\displaystyle\int_{-1}^{3}\left(2x+3-x^2\right)dx$

A. I only B. II only C. III only D. II and III E. I, II, and III

126. **(Calc)** On the graph to the right, the line $y = kx - 10$, k a constant, is tangent to the graph of $y = x^2 - 4$. Region R is enclosed by the graphs of $y = kx - 10$, $y = x^2 - 4$, and the y-axis. Find the area of region R.



A. 4.287 B. 4.899 C. 6.124

D. 9.798 E. 12.247

---

127. If the region bounded by the y-axis, the line $y = 4$, and the curve $y = \sqrt{x}$ is revolved about the x-axis, the volume of the solid generated would be

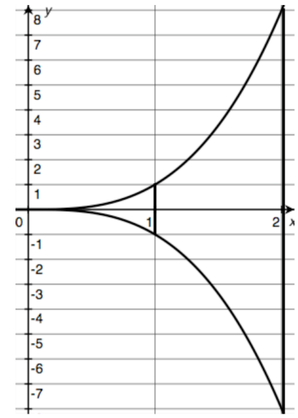
A. $\dfrac{88\pi}{3}$ B. 56π C. 128π D. $\dfrac{128\pi}{3}$ E. $\dfrac{640\pi}{3}$

---

128. **(Calc)** If the region bounded by the x-axis, the line $x = \dfrac{\pi}{2}$, and the curve $y = \sin x$ is revolved about the line $x = \dfrac{\pi}{2}$, the volume of the solid generated would be

A. 1.142 B. 1.468 C. 1.793 D. 3.142 E. 3.586

129. A piece of candy is determined by the curves $y = \pm x^3$, for $1 \le x \le 2$ as shown in the figure to the right. For this piece of candy, the cross sections perpendicular to the x-axis are equilateral triangles. What is the volume of the piece of candy in cubic units?

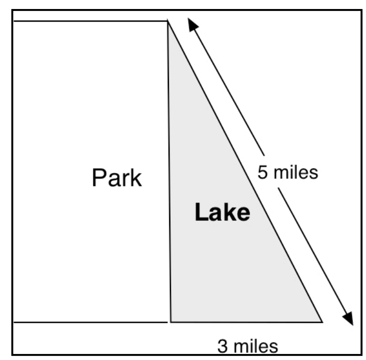


A. $\dfrac{127}{7}$ B. $\dfrac{15\sqrt{3}}{2}$ C. $\dfrac{254\sqrt{3}}{7}$

D. $\dfrac{127\sqrt{3}}{21}$ E. $\dfrac{127\sqrt{3}}{7}$

---

130. A lake shaped in a right triangle is located next to a park as shown in the figure to the right. The depth of the lake at any point along a strip x miles from the park's edge is $f(x)$ feet. Which of the following expressions gives the total volume of the lake?

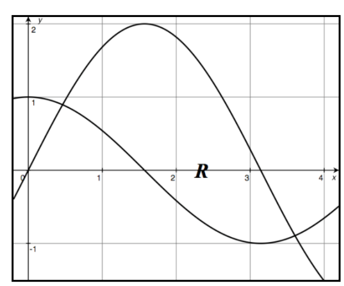


A. $\int_0^3 \left(4 - \frac{4}{3}x\right) f(x)\,dx$ B. $\int_0^4 \left(3 - \frac{3}{4}x\right) f(x)\,dx$ C. $2\int_0^3 f(x)\,dx$

D. $\int_0^4 f(x)\,dx$ E. $\frac{3}{2}\int_0^4 f(x)\,dx$

---

131. **(Calc)** Let R be the region between the graphs of $y = 2\sin x$ and $y = \cos x$ as shown in the figure to the right. The region R is the base of a solid with cross sections perpendicular to the x-axis as rectangles that are twice as high as wide. Find the volume of the solid.



A. 4.472 B. 7.854 C. 8.944

D. 15.708 E. 31.416

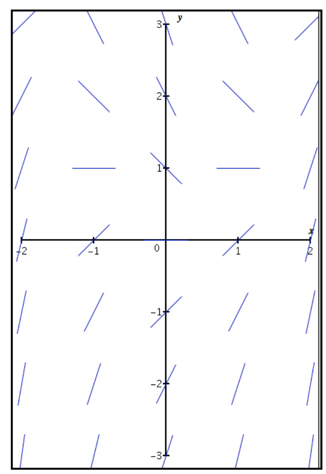
W. Differential Equations

What you are finding: A differential equation (DEQ) is in the form of $\frac{dy}{dx} = (\text{Algebraic Expression})$. The goal of solving a DEQ is to work backwards from the derivative to the function; that is to write an equation in the form of $y = f(x) + C$ (since the technique involves integration, the general solution will have a constant of integration). If the value of the function at some value of x is known (an initial condition), the value of C can be found. This is called a specific solution.

How to find it: In Calculus AB, the only types of DEQ's studied are called separable. Separable DEQ's are those than that can be in the form of $f(y)\,dy = g(x)\,dx$. Once in that form, both sides can be integrated with the constant of integration on only one side, usually the right.

Word problems involving change with respect to time are usually models of DEQ's. A favorite type is a problem using the words: The rate of change of y is proportional to some expression. The equation that describes this statement is: $\frac{dy}{dt} = k \cdot (\text{expression})$. The rate of change is usually with respect to time.
Usually there is a problem that requires you to create a slope field. Simply calculate the slopes using the given derivative formula and plot them on the given graph. Usually the slopes will be integer values.

132. The slope field for the equation in the figure to the right could be



A. $\frac{dy}{dx} = x + y^2$ B. $\frac{dy}{dx} = x - y^2$ C. $\frac{dy}{dx} = xy$

D. $\frac{dy}{dx} = x + y$ E. $\frac{dy}{dx} = x^2 - y$

---

133. If $\frac{dy}{dx} = 3x^2y^2$ and if $y = 1$ when $x = 2$, then when $x = -1$, $y =$

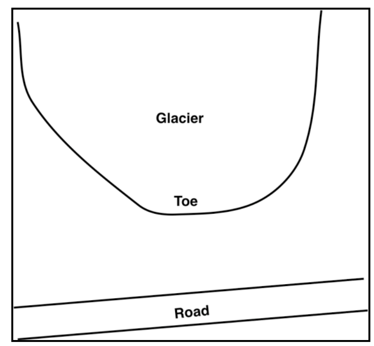
A. $\frac{1}{10}$ B. $\frac{1}{8}$ C. $\frac{1}{19}$ D. $\frac{-1}{8}$ E. -1

134. The rate of change of the volume V of the oil in a tank with respect to time t is inversely proportional to the square root of V. If the tank is empty at time $t = 0$ and $V = 9$ when $t = 2$, find the equation which describes the relationship between t and V.

A. $V = \frac{9}{4}t^2$ B. $V = \frac{t^2}{36}$ C. $V^{3/2} = \frac{4}{9}t$ D. $V^{3/2} = \frac{27}{2}t$ E. $V = 3t$

---

135. **(Calc)** The Athabasca Glacier in Canada is one of the few glaciers to which you can drive. Like many glaciers in the world, it is receding (in this case its toe is moving away from the road). In the year 2000, its toe was 500 feet from the road and in the year 2010, its toe was 600 feet from the road. The rate the distance that the toe increases from the road is proportional to that distance. Assuming the glacier recedes at the same rate, what will be the average distance in feet per year (nearest integer) the glacier recedes from 2000 to the year 2050?



A. 10 B. 13 C. 15 D. 16 E. 18