

Homework Assignment

If the radius of convergence of the power series $\sum_{n=0}^{\infty} c_n x^n$ is 10, what is the radius of convergence of the series $\sum_{n=1}^{\infty} n c_n x^{n-1}$? Why?

Suppose you know that the series $\sum_{n=0}^{\infty} b_n x^n$ converges for $|x| < 2$. What can you say about the following series? Why?

$$\sum_{n=0}^{\infty} \frac{b_n}{n+1} x^{n+1}$$

3-10 □ Find a power series representation for the function and determine the interval of convergence.

3. $f(x) = \frac{1}{1+x}$

4. $f(x) = \frac{x}{1-x}$

5. $f(x) = \frac{1}{1-x^3}$

6. $f(x) = \frac{1}{1+9x^2}$

7. $f(x) = \frac{1}{4+x^2}$

8. $f(x) = \frac{1+x^2}{1-x^2}$

9. $f(x) = \frac{1}{x-5}$

10. $f(x) = \frac{x}{4x+1}$

11-12 □ Express the function as the sum of a power series by first using partial fractions. Find the interval of convergence.

11. $f(x) = \frac{3}{x^2+x-2}$

12. $f(x) = \frac{7x-1}{3x^2+2x-1}$

13-20 □ Find a power series representation for the function and determine the radius of convergence.

13. $f(x) = \frac{1}{(1+x)^2}$

14. $f(x) = \ln(1+x)$

15. $f(x) = \frac{1}{(1+x)^3}$

16. $f(x) = x \ln(1+x)$

17. $f(x) = \ln(5-x)$

18. $f(x) = \frac{x^2}{(1-2x)^2}$

19. $f(x) = \frac{x^3}{(x-2)^2}$

20. $f(x) = \arctan(x/3)$

21-24 □ Find a power series representation for f , and graph f and several partial sums $s_n(x)$ on the same screen. What happens as n increases?

21. $f(x) = \ln(3+x)$

22. $f(x) = \frac{1}{x^2+25}$

23. $f(x) = \ln\left(\frac{1+x}{1-x}\right)$

24. $f(x) = \tan^{-1}(2x)$

25-28 □ Evaluate the indefinite integral as a power series.

25. $\int \frac{1}{1+x^4} dx$

26. $\int \frac{x}{1+x^5} dx$

27. $\int \frac{\arctan x}{x} dx$

28. $\int \tan^{-1}(x^2) dx$

29-32 □ Use a power series to approximate the definite integral to six decimal places.

29. $\int_0^{0.2} \frac{1}{1+x^4} dx$

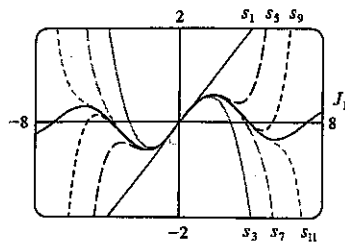
30. $\int_0^{1/2} \tan^{-1}(x^2) dx$

31. $\int_0^{1/3} x^2 \tan^{-1}(x^4) dx$

32. $\int_0^{0.5} \frac{dx}{1+x^6}$

Answers on next page.

(b), (c)



35. $(-1, 1), f(x) = (1 + 2x)/(1 - x^2)$ 39. 2

Exercises 12.9 □ page 783

1. 10 3. $\sum_{n=0}^{\infty} (-1)^n x^n, (-1, 1)$ 5. $\sum_{n=0}^{\infty} x^{3n}, (-1, 1)$

7. $\sum_{n=0}^{\infty} \frac{(-1)^n}{4^{n+1}} x^{2n}, (-2, 2)$ 9. $-\sum_{n=0}^{\infty} \frac{1}{5^{n+1}} x^n, (-5, 5)$

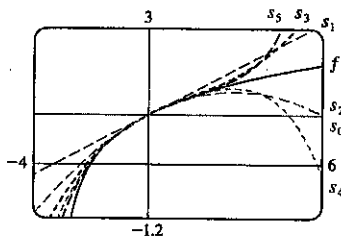
11. $\sum_{n=0}^{\infty} \left[\frac{(-1)^{n+1}}{2^{n+1}} - 1 \right] x^n, (-1, 1)$

13. $\sum_{n=0}^{\infty} (-1)^n (n+1) x^n, R = 1$

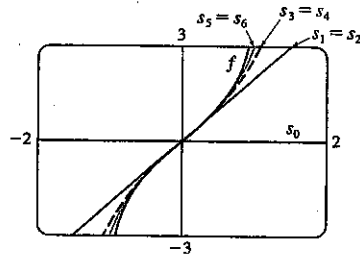
15. $\frac{1}{2} \sum_{n=0}^{\infty} (-1)^n (n+2)(n+1) x^n, R = 1$

17. $\ln 5 - \sum_{n=1}^{\infty} \frac{x^n}{n5^n}, R = 5$ 19. $\sum_{n=3}^{\infty} \frac{n-2}{2^{n-1}} x^n, (-2, 2)$

21. $\ln 3 + \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n3^n} x^n, R = 3$



23. $\sum_{n=0}^{\infty} \frac{2x^{2n+1}}{2n+1}, R = 1$



25. $C + \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+1}}{4n+1}$ 27. $C + \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)^2}$

29. 0.199936 31. 0.000065 33. 0.09531

35. (b) 0.920 39. $[-1, 1], [-1, 1), (-1, 1)$

Exercises 12.10 □ page 794

1. $b_8 = f^{(8)}(5)/8!$ 3. $\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}, R = \infty$

5. $\sum_{n=0}^{\infty} (-1)^n \frac{(n+1)(n+2)}{2} x^n, R = 1$

7. $\sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!}, R = \infty$ 9. $7 + 5(x-2) + (x-2)^2, R = \infty$

11. $\sum_{n=0}^{\infty} \frac{e^3}{n!} (x-3)^n, R = \infty$ 13. $\sum_{n=0}^{\infty} (-1)^n (x-1)^n, R = 1$

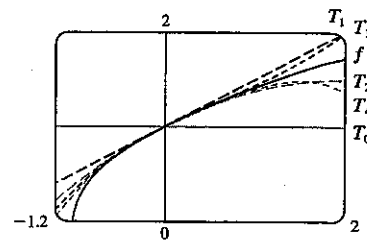
15. $\frac{\sqrt{2}}{2} \sum_{n=0}^{\infty} (-1)^n \left[\frac{1}{(2n)!} \left(x - \frac{\pi}{4}\right)^{2n} + \frac{1}{(2n+1)!} \left(x - \frac{\pi}{4}\right)^{2n+1} \right], R = \infty$

21. $\sum_{n=0}^{\infty} (-1)^n \frac{n^{2n}}{(2n)!} x^{2n}$ 23. $\sum_{n=0}^{\infty} (-1)^n \frac{1}{2n+1} x^{2n+2}$

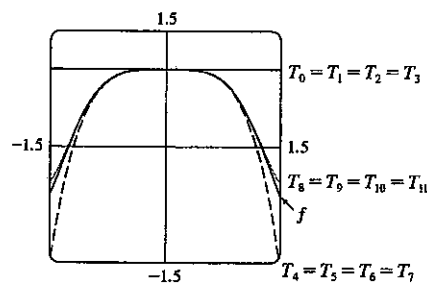
25. $\sum_{n=0}^{\infty} (-1)^n \frac{1}{n!} x^{n+2}$ 27. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} 2^{2n-1} x^{2n}}{(2n)!}$

29. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n+1)!}$

31. $1 + \frac{x}{2} + \sum_{n=2}^{\infty} (-1)^{n-1} \frac{1 \cdot 3 \cdot 5 \cdot \dots \cdot (2n-3)}{2^n n!} x^n, R = 1$



33. $\sum_{n=0}^{\infty} (-1)^n \frac{1}{(2n)!} x^{4n}, R = \infty$ 35. $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n}, 0.09531$



37. $C + \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+3}}{(4n+3)(2n+1)!}$

39. $C + x + \frac{x^4}{8} + \sum_{n=2}^{\infty} (-1)^{n-1} \frac{1 \cdot 3 \cdot 5 \cdot \dots \cdot (2n-3)}{2^n n! (3n+1)} x^{3n+1}$

41. 0.310 43. 0.09998750 45. $\frac{1}{3}$ 47. $\frac{1}{120}$

49. $1 - \frac{3}{2}x^2 + \frac{25}{24}x^4$ 51. $-x + \frac{1}{2}x^2 - \frac{1}{3}x^3$ 53. e^{-x^4}

55. $1/\sqrt{2}$ 57. $e^3 - 1$

Exercises 12.11 □ page 798

1. $1 + \frac{x}{2} + \sum_{n=2}^{\infty} (-1)^{n-1} \frac{1 \cdot 3 \cdot 5 \cdot \dots \cdot (2n-3)}{2^n n!} x^n, R = 1$

3. $\sum_{n=0}^{\infty} (-1)^n \frac{(n+1)(n+2)}{2^{n+4}} x^n, R = 2$

5. $1 - 2x - \sum_{n=2}^{\infty} \frac{3 \cdot 7 \cdot \dots \cdot (4n-5) \cdot 2^n}{n!} x^n, R = \frac{1}{8}$