Multiple-Choice Questions on the Fundamental Theorem of Calculus

1. 1969 BC12

If $F(x)=\int_0^x e^{-t^2}\,dt$, then $F'(x)=$

(A) $2xe^{-x^2}$ (B) $-2xe^{-x^2}$ (C) $\dfrac{e^{-x^2+1}}{-x^2+1}-e$ (D) $e^{-x^2}-1$ (E) e^{-x^2}

2. 1969 BC22

If $f(x)=\int_0^x \dfrac{1}{\sqrt{t^3+2}}\,dt$, which of the following is FALSE?

(A) $f(0)=0$
(B) f is continuous at x for all $x\geq 0$
(C) $f(1)>0$
(D) $f'(1)=\dfrac{1}{\sqrt{3}}$
(E) $f(-1)>0$

3. 1973 AB20

If F and f are continuous functions such that $F'(x)=f(x)$ for all x, then $\int_a^b f(x)\,dx$ is

(A) $F'(a)-F'(b)$
(B) $F'(b)-F'(a)$
(C) $F(a)-F(b)$
(D) $F(b)-F(a)$
(E) none of the above

4. 1973 BC45

Suppose $g'(x)<0$ for all $x\geq 0$ and $F(x)=\int_0^x t\,g'(t)\,dt$ for all $x\geq 0$. Which of the following statements is FALSE?

(A) F takes on negative values.
(B) F is continuous for all $x>0$.
(C) $F(x)=x\,g(x)-\int_0^x g(t)\,dt$
(D) $F'(x)$ exists for all $x>0$.
(E) F is an increasing function.

Special Focus: The Fundamental Theorem of Calculus

5. 1985 AB42

$$\frac{d}{dx}\int_2^x \sqrt{1+t^2}\,dt =$$

(A) $\dfrac{x}{\sqrt{1+x^2}}$ (B) $\sqrt{1+x^2}-5$ (C) $\sqrt{1+x^2}$ (D) $\dfrac{x}{\sqrt{1+x^2}}-\dfrac{1}{\sqrt{5}}$

(E) $\dfrac{1}{2\sqrt{1+x^2}}-\dfrac{1}{2\sqrt{5}}$

6. 1988 AB13

If the function f has a continuous derivative on $[0,c]$, then $\int_0^c f'(x)\,dx =$

(A) $f(c)-f(0)$ (B) $\left|f(c)-f(0)\right|$ (C) $f(c)$ (D) $f(x)+c$

(E) $f''(c)-f''(0)$

7. 1988 AB25

For all $x>1$, if $f(x)=\int_1^x \frac{1}{t}\,dt$, then $f'(x)=$

(A) 1 (B) $\dfrac{1}{x}$ (C) $\ln x-1$ (D) $\ln x$ (E) e^x

8. 1988 BC14

If $F(x)=\int_1^{x^2} \sqrt{1+t^3}\,dt$, then $F'(x)=$

(A) $2x\sqrt{1+x^6}$ (B) $2x\sqrt{1+x^3}$ (C) $\sqrt{1+x^6}$ (D) $\sqrt{1+x^3}$

(E) $\int_1^{x^2} \dfrac{3t^2}{2\sqrt{1+t^3}}\,dt$

9. 1993 AB41

$\dfrac{d}{dx}\int_0^x \cos(2\pi u)\,du$ is

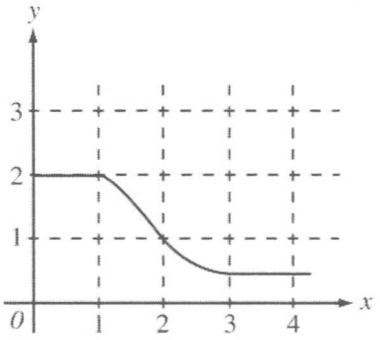
(A) 0 (B) $\dfrac{1}{2\pi}\sin x$ (C) $\dfrac{1}{2\pi}\cos(2\pi x)$ (D) $\cos(2\pi x)$ (E) $2\pi\cos(2\pi x)$

10. 1993 BC41

Let $f(x)=\int_{-2}^{x^2-3x} e^{t^2}\,dt$. At what value of x is $f(x)$ a minimum?

(A) For no value of x (B) $\frac{1}{2}$ (C) $\frac{3}{2}$ (D) 2 (E) 3

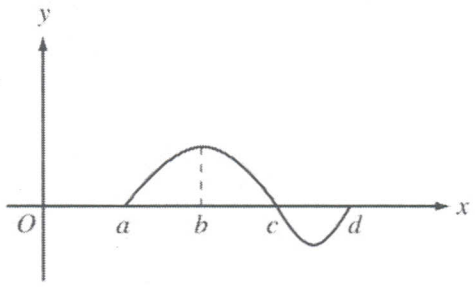
11. 1997 AB78



The graph of f is shown in the figure above. If $\int_1^3 f(x)\,dx=2.3$ and $F'(x)=f(x)$, then $F(3)-F(0)=$

(A) 0.3 (B) 1.3 (C) 3.3 (D) 4.3 (E) 5.3

12. 1997 BC22



The graph of f is shown in the figure above. If $g(x)=\int_a^x f(t)\,dt$, for what value of x does $g(x)$ have a maximum?

(A) a (B) b (C) c (D) d
(E) It cannot be determined from the information given.

Special Focus: The Fundamental Theorem of Calculus

13. 1997 BC88

Let $f(x) = \int_0^{x^2} \sin t \, dt$. At how many points in the closed interval $\left[0, \sqrt{\pi}\right]$ does the instantaneous rate of change of f equal the average rate of change of f on that interval?

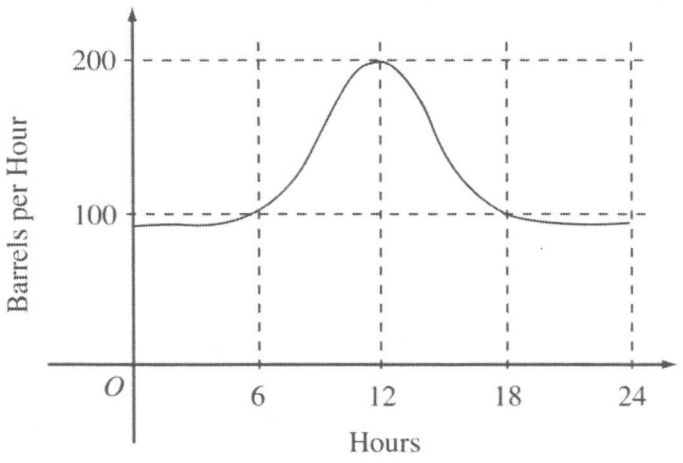
(A) Zero
(B) One
(C) Two
(D) Three
(E) Four

14. 1997 BC89

If f is the antiderivative of $\dfrac{x^2}{1+x^5}$ such that $f(1) = 0$, then $f(4) =$

(A) -0.012 (B) 0 (C) 0.016 (D) 0.376 (E) 0.629

15. 1998 AB9



The flow of oil, in barrels per hour, through a pipeline on July 9 is given by the graph shown above. Of the following, which best approximates the total number of barrels of oil that passed through the pipeline that day?

(A) 500 (B) 600 (C) 2,400 (D) 3,000 (E) 4,800

16. 1998 AB11

If f is a linear function and $0 < a < b$, then $\int_a^b f''(x)\,dx =$

(A) 0 (B) 1 (C) $\frac{ab}{2}$ (D) $b-a$ (E) $\frac{b^2-a^2}{2}$

17. 1998 AB15

If $F(x) = \int_0^x \sqrt{t^3+1}\,dt$, then $F'(2) =$

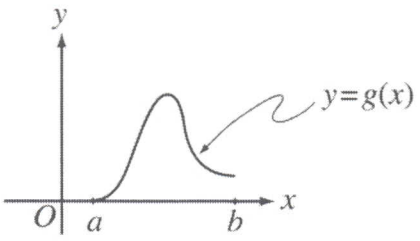
(A) –3 (B) –2 (C) 2 (D) 3 (E) 18

18. 1998 AB88

Let $F(x)$ be an antiderivative of $\frac{(\ln x)^3}{x}$. If $F(1) = 0$ then $F(9) =$

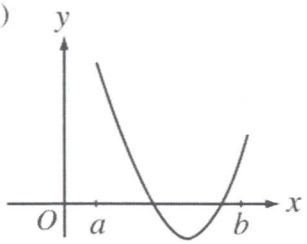
(A) 0.048 (B) 0.144 (C) 5.827 (D) 23.308 (E) 1,640.250

19. 1998 BC88

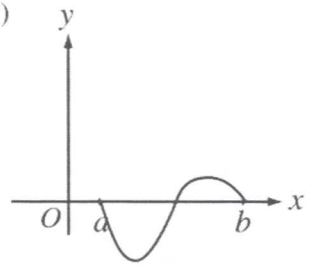


Let $g(x) = \int_a^x f(t)\,dt$, where $a \le x \le b$. The figure above shows the graph of g on $[a,b]$. Which of the following could be the graph of f on $[a,b]$?

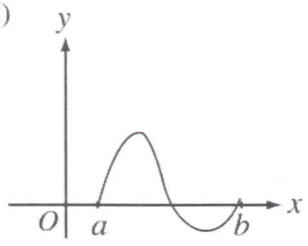
(A)



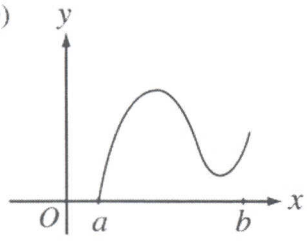
(B)



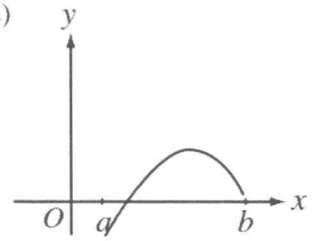
(C)



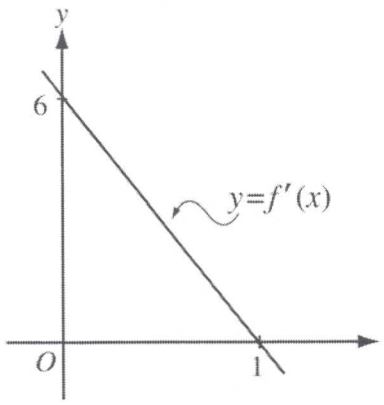
(D)



(E)



20. 2003 AB22



The graph of f', the derivative of f, is the line shown in the figure above. If $f(0) = 5$, then $f(1) =$

(A) 0 (B) 3 (C) 6 (D) 8 (E) 11

21. 2003 AB82/BC82

The rate of change of the altitude of a hot-air balloon is given by $r(t) = t^3 - 4t^2 + 6$ for $0 \le t \le 8$. Which of the following expressions gives the change in altitude of the balloon during the time the altitude is decreasing?

(A) $\int_{1.572}^{3.514} r(t)\,dt$

(B) $\int_{0}^{8} r(t)\,dt$

(C) $\int_{0}^{2.667} r(t)\,dt$

(D) $\int_{1.572}^{3.514} r'(t)\,dt$

(E) $\int_{0}^{2.667} r'(t)\,dt$

Special Focus: The Fundamental Theorem of Calculus

22. 2003 AB84

A pizza, heated to a temperature of 350 degrees Fahrenheit (°F) is taken out of an oven and placed in a 75°F room at time $t = 0$ minutes. The temperature of the pizza is changing at a rate of $-110e^{-0.4t}$ degrees Fahrenheit per minute. To the nearest degree, what is the temperature of the pizza at time $t = 5$ minutes?

(A) 112°F (B) 119°F (C) 147°F (D) 238°F (E) 335°F

23. 2003 AB91

A particle moves along the x-axis so that at any time $t > 0$, its acceleration is given by $a(t) = \ln\left(1 + 2^t\right)$. If the velocity of the particle is 2 at time $t = 1$ then the velocity of the particle at time $t = 2$ is

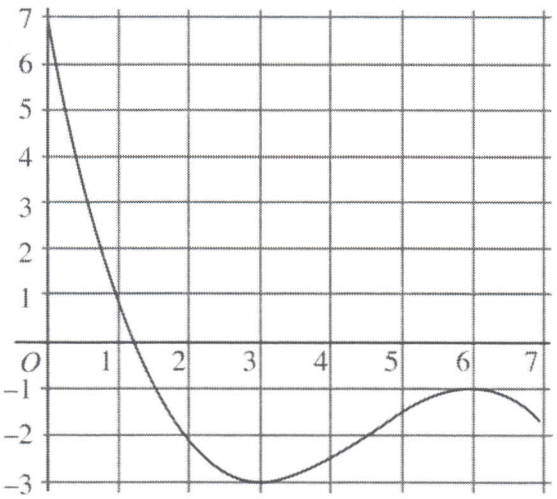
(A) 0.462 (B) 1.609 (C) 2.555 (D) 2.886 (E) 3.346

24. 2003 AB92

Let g be the function given by $g(x) = \int_0^x \sin\left(t^2\right) dt$ for $-1 \le x \le 3$. On which of the following intervals is g decreasing?

(A) $-1 \le x \le 0$
(B) $0 \le x \le 1.772$
(C) $1.253 \le x \le 2.171$
(D) $1.772 \le x \le 2.507$
(E) $2.802 \le x \le 3$

25. 2003 BC18



Graph of f

The graph of the function f shown in the figure above has horizontal tangents at $x = 3$ and $x = 6$. If $g(x) = \int_0^{2x} f(t)\,dt$, what is the value of $g'(3)$?

(A) 0 (B) −1 (C) −2 (D) −3 (E) −6

26. 2003 BC27

$$\frac{d}{dx}\left(\int_0^{x^3} \ln\left(t^2+1\right)dt\right) =$$

(A) $\dfrac{2x^3}{x^6+1}$ (B) $\dfrac{3x^2}{x^6+1}$ (C) $\ln\left(x^6+1\right)$ (D) $2x^3\ln\left(x^6+1\right)$

(E) $3x^2\ln\left(x^6+1\right)$

Special Focus: The Fundamental Theorem of Calculus

27. 2003 BC80

Insects destroyed a crop at the rate of $\frac{100e^{-0.1t}}{2-e^{-3t}}$ tons per day, where time t is measured in days. To the nearest ton, how many tons did the insects destroy during the time interval $7 \le t \le 14$?

(A) 125 (B) 100 (C) 88 (D) 50 (E) 12

28. 2003 BC87

A particle moves along the x-axis so that at any time $t \ge 0$, its velocity is given by $v(t) = \cos\left(2 - t^2\right)$. The position of the particle is 3 at time $t = 0$. What is the position of the particle when its velocity is first equal to 0?

(A) 0.411 (B) 1.310 (C) 2.816 (D) 3.091 (E) 3.411