COMMON CORE ASSESSMENT COMPARISON FOR MATHEMATICS

NUMBER AND QUANTITY GRADES 9–11

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Table of Contents

INTRODUCTION

The purpose of this document is to illustrate the differences between the Delaware Comprehensive Assessment System (DCAS) and the expectations of the next-generation Common Core State Standard (CCSS) assessment in Mathematics. A side-by-side comparison of the current design of an operational assessment item and the expectations for the content and rigor of a next-generation Common Core mathematical item are provided for each CCSS. The samples provided are designed to help Delaware's educators better understand the instructional shifts needed to meet the rigorous demands of the CCSS. This document does not represent the test specifications or blueprints for each grade level, for DCAS, or the next-generation assessment.

For mathematics, next-generation assessment items were selected for CCSS that represent the shift in content at the new grade level. Sites used to select the next-generation assessment items include:

- Smarter Balanced Assessment Consortium
- Partnership of Assessment of Readiness for College and Career
- Illustrative Mathematics
- Mathematics Assessment Project

Using released items from other states, a DCAS-like item, aligned to the same CCSS, was chosen. These examples emphasize the contrast in rigor between the previous Delaware standards, known as Grade-Level Expectations, and the Common Core State Standards.

Section 1, DCAS-Like and Next-Generation Assessment Comparison, includes content that is in the CCSS at a different "rigor" level. The examples are organized by the CCSS. For some standards, more than one example may be given to illustrate the different components of the standard. Additionally, each example identifies the standard and is separated into two parts. Part A is an example of a DCAS-like item, and Part B is an example of a next-generation item based on CCSS.

Section 2, Standards Not DCAS-Tested, includes items whose CCSS content is new to the grade level and therefore not on DCAS. A sample next-generation item may have been included. For some grades, all CCSS may not be illustrated.

Section 3 includes at least one Performance Task that addresses multiple aspects of the CCSS (content and mathematical practices).

How to Use Various Aspects of This Document

- Analyze the way mathematics standards are conceptualized in each item or task.
- Identify the instructional shifts that need to occur to prepare students to address these more rigorous demands. Develop a plan to implement the necessary instructional changes.
- Develop awareness to the way strong distracters are presented in each multiple-choice item.
- Notice how numbers (e.g., fractions instead of whole numbers) are used in the sample items.

- Recognize that the sample items and tasks are only one way of assessing the standard.
- Understand that the sample items and tasks do not represent a mini-version of the next-generation assessment.
- Instruction should address “focus,” coherence,” and “rigor” of mathematics concepts.
- Instruction should embed mathematical practices when teaching mathematical content.

Your feedback is welcomed. Please do not hesitate to contact Katia Foret at katia.foret@doe.k12.de.us or Rita Fry at rita.fry@doe.k12.de.us with suggestions, questions, and/or concerns.

* The Smarter Balanced Assessment Consortium has a 30-item practice test available for each grade level (3-8 and 11) for mathematics and ELA (including reading, writing, listening, and research). These practice tests allow students to experience items that look and function like those being developed for the Smarter Balanced assessments. The practice test also includes performance tasks and is constructed to follow a test blueprint similar to the blueprint intended for the operational test. The Smarter Balanced site is located at: http://www.smarterbalanced.org/.

Priorities in Mathematics

Grade	Priorities in Support of Rich Instruction and Expectations of Fluency and Conceptual Understanding
K–2	Addition and subtraction, measurement using whole number quantities
3–5	Multiplication and division of whole numbers and fractions
6	Ratios and proportional reasoning; early expressions and equations
7	Ratios and proportional reasoning; arithmetic of rational numbers
8	Linear algebra

Common Core State Standards for Mathematical Practices

		Student Dispositions:	Teacher Actions to Engage Students in Practices:
Essential Processes for a Productive Math Thinker	**1. Make sense of problems and persevere in solving them**	▪ Have an understanding of the situation ▪ Use patience and persistence to solve problem ▪ Be able to use different strategies ▪ Use self-evaluation and redirections ▪ Communicate both verbally and written ▪ Be able to deduce what is a reasonable solution	▪ Provide open-ended and rich problems ▪ Ask probing questions ▪ Model multiple problem-solving strategies through Think-Aloud ▪ Promote and value discourse ▪ Integrate cross-curricular materials ▪ Promote collaboration ▪ Probe student responses (correct or incorrect) for understanding and multiple approaches ▪ Provide scaffolding when appropriate ▪ Provide a safe environment for learning from mistakes
	6. Attend to precision	▪ Communicate with precision—orally and written ▪ Use mathematics concepts and vocabulary appropriately ▪ State meaning of symbols and use them appropriately ▪ Attend to units/labeling/tools accurately ▪ Carefully formulate explanations and defend answers ▪ Calculate accurately and efficiently ▪ Formulate and make use of definitions with others ▪ Ensure reasonableness of answers ▪ Persevere through multiple-step problems	▪ Encourage students to think aloud ▪ Develop explicit instruction/teacher models of thinking aloud ▪ Include guided inquiry as teacher gives problem, students work together to solve problems, and debrief time for sharing and comparing strategies ▪ Use probing questions that target content of study ▪ Promote mathematical language ▪ Encourage students to identify errors when answers are wrong
Reasoning and Explaining	**2. Reason abstractly and quantitatively**	▪ Create multiple representations ▪ Interpret problems in contexts ▪ Estimate first/answer reasonable ▪ Make connections ▪ Represent symbolically ▪ Talk about problems, real-life situations ▪ Attend to units ▪ Use context to think about a problem	▪ Develop opportunities for problem-solving strategies ▪ Give time for processing and discussing ▪ Tie content areas together to help make connections ▪ Give real-world situations ▪ Demonstrate thinking aloud for students' benefit ▪ Value invented strategies and representations ▪ More emphasis on the process instead of on the answer
	3. Construct viable arguments and critique the reasoning of others	▪ Ask questions ▪ Use examples and counter examples ▪ Reason inductively and make plausible arguments ▪ Use objects, drawings, diagrams, and actions ▪ Develop ideas about mathematics and support their reasoning ▪ Analyze others arguments ▪ Encourage the use of mathematics vocabulary	▪ Create a safe environment for risk-taking and critiquing with respect ▪ Provide complex, rigorous tasks that foster deep thinking ▪ Provide time for student discourse ▪ Plan effective questions and student grouping ▪ Probe students

Mathematical Practices		Students:	Teacher(s) promote(s) by:
Modeling and Using Tools	**4. Model with mathematics**	▪ Realize that mathematics (numbers and symbols) is used to solve/work out real-life situations ▪ Analyze relationships to draw conclusions ▪ Interpret mathematical results in context ▪ Show evidence that they can use their mathematical results to think about a problem and determine if the results are reasonable—if not, go back and look for more information ▪ Make sense of the mathematics	▪ Allowing time for the process to take place (model, make graphs, etc.) ▪ Modeling desired behaviors (think alouds) and thought processes (questioning, revision, reflection/written) ▪ Making appropriate tools available ▪ Creating an emotionally safe environment where risk-taking is valued ▪ Providing meaningful, real-world, authentic, performance-based tasks (non-traditional work problems) ▪ Promoting discourse and investigations
	5. Use appropriate tools strategically	▪ Choose the appropriate tool to solve a given problem and deepen their conceptual understanding (paper/pencil, ruler, base ten blocks, compass, protractor) ▪ Choose the appropriate technological tool to solve a given problem and deepen their conceptual understanding (e.g., spreadsheet, geometry software, calculator, web 2.0 tools) ▪ Compare the efficiency of different tools ▪ Recognize the usefulness and limitations of different tools	▪ Maintaining knowledge of appropriate tools ▪ Modeling effectively the tools available, their benefits, and limitations ▪ Modeling a situation where the decision needs to be made as to which tool should be used ▪ Comparing/contrasting effectiveness of tools ▪ Making available and encouraging use of a variety of tools
Seeing Structure and Generalizing	**7. Look for and make use of structure**	▪ Look for, interpret, and identify patterns and structures ▪ Make connections to skills and strategies previously learned to solve new problems/tasks independently and with peers ▪ Reflect and recognize various structures in mathematics ▪ Breakdown complex problems into simpler, more manageable chunks ▪ "Step back" or shift perspective ▪ Value multiple perspectives	▪ Being quiet and structuring opportunities for students to think aloud ▪ Facilitating learning by using open-ended questions to assist students in exploration ▪ Selecting tasks that allow students to discern structures or patterns to make connections ▪ Allowing time for student discussion and processing in place of fixed rules or definitions ▪ Fostering persistence/stamina in problem solving ▪ Allowing time for students to practice
	8. Look for and express regularity in repeated reasoning	▪ Identify patterns and make generalizations ▪ Continually evaluate reasonableness of intermediate results ▪ Maintain oversight of the process ▪ Search for and identify and use shortcuts	▪ Providing rich and varied tasks that allow students to generalize relationships and methods and build on prior mathematical knowledge ▪ Providing adequate time for exploration ▪ Providing time for dialogue, reflection, and peer collaboration ▪ Asking deliberate questions that enable students to reflect on their own thinking ▪ Creating strategic and intentional check-in points during student work time

For classroom posters depicting the Mathematical Practices, please see: http://seancarberry.cmswiki.wikispaces.net/file/detail/12-20math.docx

The Real Number System (N.RN)

Specific modeling standards appear throughout the high school mathematical standards and are indicated by an asterisk (*).

Cluster: Extend the properties of exponents to rational exponents.

9-11.N.RN.1 – Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define 51/3 to be the cube root of 5 because we want (51/3)3 = 5(1/3)3 to hold, so (51/3)3 must equal 5.*

DCAS-Like

1A

Which expression is equivalent to $\left(\sqrt{2x^2}\right)^4$?

A. $2x^4$

B. $4x^4$

C. $4x^8$

D. $8x^8$

Next-Generation

1B

Select **Yes** or **No** to all alternate forms of $\frac{\sqrt{x}}{x^2}$

a. $x^{-\frac{3}{2}}$ ○ Yes ○ No

b. $\frac{1}{\sqrt{x^3}}$ ○ Yes ○ No

c. $\sqrt{\frac{x}{3}}$ ○ Yes ○ No

d. $\sqrt[3]{x}$ ○ Yes ○ No

e. $\frac{1}{x\sqrt{x}}$ ○ Yes ○ No

9-11.N.RN.2 – Rewrite expressions involving radicals and rational exponents using the properties of exponents.

DCAS-Like

2A

Simplify: $\sqrt{16} + \sqrt[3]{8}$

A. 4

B. 6

C. 9

D. 10

Next-Generation

2B

For items a. through e., determine whether each equation is **True** or **False**.

a. $\sqrt{32} = 2^{\frac{5}{2}}$ ○ True ○ False

b. $16^{\frac{3}{2}} = 8^2$ ○ True ○ False

c. $4^{\frac{1}{2}} = \sqrt[4]{64}$ ○ True ○ False

d. $2^8 = (\sqrt[3]{16})^6$ ○ True ○ False

e. $(\sqrt{64})^{\frac{1}{3}} = 8^{\frac{1}{6}}$ ○ True ○ False

9-11.N.RN.2 – Rewrite expressions involving radicals and rational exponents using the properties of exponents.

DCAS-Like

3A

Which value of x makes this equation true?

$$9(x-7)^{\frac{4}{3}} = 9$$

A. 1

B. 7

C. 8

D. 34

Next-Generation

3B

In each of the following problems, a number is given. If possible, determine whether the given number is rational or irrational. In some cases, it may be impossible to determine whether the given number is rational or irrational.

a.	$4+\sqrt{7}$	○ True	○ False	○ Impossible
b.	$\frac{\sqrt{45}}{\sqrt{5}}$	○ True	○ False	○ Impossible
c.	$\frac{6}{\pi}$	○ True	○ False	○ Impossible
d.	$\sqrt{2}+\sqrt{3}$	○ True	○ False	○ Impossible
e.	$\frac{2+\sqrt{7}}{2a+\sqrt{7a^2}}$, where a is a positive integer	○ True	○ False	○ Impossible
f.	$x+y$, where x and y are irrational numbers	○ True	○ False	○ Impossible

Cluster: Use properties of rational and irrational numbers.

9-11.N.RN.3 – Explain why the sum or product of rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

DCAS-Like

4A

Do either of the items below contradict the statement: *"The sum of two rational numbers is a rational number"*?

Item A: $\frac{2}{5}+\frac{1}{3}=\frac{11}{15}$

Item B: $\frac{2}{5}+\frac{3}{5}=1$

A. Neither Item A nor Item B contradicts the given statement.

B. Both Item A and Item B contradicts the given statement.

C. Item A contradicts the statement. Item B is an example where the statement is true.

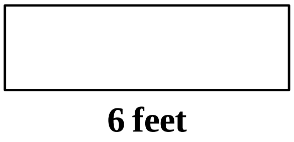
D. Item B contradicts the statement. Item A is an example where the statement is true.

Next-Generation

4B

Part A

The rectangle shown below has a length of 6 feet.



The value of the area of the rectangle, in square feet, is an irrational number. Therefore, the number that represents the width of the rectangle must be:

a. A whole number

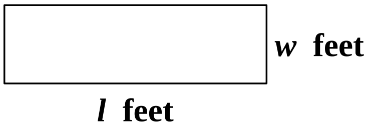
b. A rational number

c. An irrational number

d. A non-real complex number

Part B

The length, l, and width, w, of the rectangle shown below have values that are rational numbers.



Construct an informal proof that shows that the value of the area, in square feet, of the rectangle must be a rational number.

9-11.N.RN.3 – Explain why the sum or product of rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

DCAS-Like

5A

Which of the following assertions is true?

A. Any integer is rational.

B. Between any two distinct rational numbers there is a finite number of other rational numbers.

C. Rational numbers are closed under addition, subtraction, and multiplication, but NOT division.

D. The representation of a rational number as a fraction is unique.

Next-Generation

5B

a. Complete the addition table.

$+$	5	$\frac{1}{2}$	0	$\sqrt{2}$	$-\sqrt{2}$	π
5	10	5.5				
$\frac{1}{2}$						
0						
$\sqrt{2}$						
$-\sqrt{2}$						
π						

b. Complete the multiplication table.

$\times$	5	$\frac{1}{2}$	0	$\sqrt{2}$	$\frac{1}{-\sqrt{2}}$	π
5	25	2.5				
$\frac{1}{2}$						
0						
$\sqrt{2}$						
$\frac{1}{-\sqrt{2}}$						
π						

Continue to next page for item c.

c. Based on the above information, conjecture which of the statements is ALWAYS true, which is SOMETIMES true, and which is NEVER true.

	Response	**Response Choices**
1. The sum of a rational number and a rational number is rational.		▪ Always true ▪ Sometimes true ▪ Never true
2. The sum of a rational number and an irrational number is irrational.		▪ Always true ▪ Sometimes true ▪ Never true
3. The sum of an irrational number and an irrational number is irrational.		▪ Always true ▪ Sometimes true ▪ Never true
4. The product of a rational number and a rational number is rational.		▪ Always true ▪ Sometimes true ▪ Never true
5. The product of a rational number and an irrational number is irrational.		▪ Always true ▪ Sometimes true ▪ Never true
6. The product of an irrational number and an irrational number is irrational.		▪ Always true ▪ Sometimes true ▪ Never true

Quantities (N.Q)

Specific modeling standards appear throughout the high school mathematical standards and are indicated by an asterisk (*).

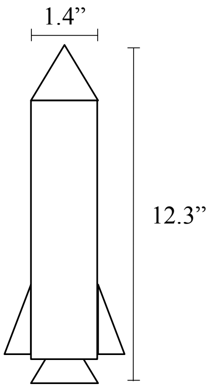
Cluster: Reason quantitatively and use units to solve problems.

9-11.N.Q.1 – Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.*

DCAS-Like

6A

The scale drawing for the rocket was drawn incorrectly. The actual rocket is supposed to be 15 feet in diameter and 138 feet high. What should the measurements on the drawing be?



1 inch = 12 feet

A. 1.25 and 11.5”

B. 1.3” and 11.6”

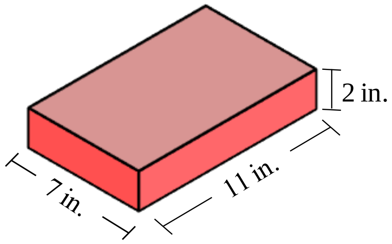
C. 1.25” and 12.3”

D. 1.4” and 11.5”

Next-Generation

6B

Hannah makes 6 cups of cake batter. She pours and levels all the batter into a rectangular cake pan with a length of 11 inches, a width of 7 inches, and a depth of 2 inches.



One cubic inch is approximately equal to 0.069 cups. What is the **depth** of the batter in the pan when it is completely poured in? Round your answer to the nearest $\frac{1}{8}$ of an inch.

9-11.N.Q.2 – Define appropriate quantities for the purpose of descriptive modeling.*

DCAS-Like

7A

Nick is in the seventh grade. He works part time as a caddy and babysits his little cousin. He has determined that it is only 4 more years until he can drive, and he wants to save for a car. He knows that it will be a used car, and that it will not cost more than $8,000. **About** how much should he plan to save each month?

A. $20

B. $50

C. $150

D. $400

Next-Generation

7B

Jan estimates that the faucet in her kitchen drips at a rate of 1 drop every 2 seconds.

a. Estimate how many times the faucet drips in a week. ____________________

Show your calculations.

Jan estimates that approximately 575 drops fill a 100 milliliter bottle.

b. Estimate how much water her leaky faucet wastes in a year. ____________________

Show your calculations.

9-11.N.Q.3 – Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.*

DCAS-Like

8A

Lisa will make punch that is 25% fruit juice by adding pure fruit juice to a 2-liter mixture that is 10% pure fruit juice. How many liters of pure fruit juice does she need to add?

A. 0.4 liter

B. 0.5 liter

C. 2 liters

D. 8 liters

Next-Generation

8B

A liquid weed-killer comes in four different bottles, all with the same active ingredient. The accompanying table gives information about the concentration of active ingredient in the bottles, the size of the bottles, and the price of the bottles. Each bottle's contents is made up of active ingredient and water.

	Concentration	Amount in Bottle	Price of Bottle
A	1.04%	64 fl oz	$12.99
B	18.00%	32 fl oz	$22.99
C	41.00%	32 fl oz	$39.99
D	1.04%	24 fl oz	$5.99

a. You need to apply a 1% solution of the weed-killer to your lawn. Rank the four bottles in order of best to worst buy. How did you decide what made a bottle a better buy than another?

b. The size of your lawn requires a total of 14 fl oz of active ingredient. Approximately how much would you need to spend if you bought only the A bottles? Only the B bottles? Only the C bottles? Only the D bottles?

Supposing you can only buy one type of bottle, which type should you buy so that the total cost to you is the least for this particular application of weed-killer?

The Complex Number System (N.CN)

Specific modeling standards appear throughout the high school mathematical standards and are indicated by an asterisk (*).

Cluster: Perform arithmetic operations with complex numbers.

9-11.N.CN.1 – Know there is a complex number *i* such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.

DCAS-Like

9A

Multiply: $(3 + 5i)(3 - 5i)$

A. $9 + 25i$

B. $9 - 25i$

C. 34

D. 25

Next-Generation

9B

For items a. through d. below, select **Yes** or **No** to indicate whether the equation is correct.

a. $-4i \bullet 5i = 20$ ○ Yes ○ No

b. $(7 - 6i)(-8 + 3i) = 38 - 69i$ ○ Yes ○ No

c. $(8 - 3i)^2 = 55 + 48i$ ○ Yes ○ No

d. $-6i(8 - 6i)(-8 - 8i) = -96 + 672i$ ○ Yes ○ No

9-11.N.CN.2 – Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

DCAS-Like

10A

Which expression is equivalent to $(6 - 2i)(2 - 3i)$?

A. $6 + 22i$

B. $18 + 22i$

C. $18 - 22i$

D. $6 - 22i$

Next-Generation

10B

Simplify the following expression:

$$(3 - 2i)(-7 + 4i)$$

1. Enter each step of your answer in each text box without spaces.
2. Use the symbol ^ on a keyboard to properly designate an exponent.
3. Your answer can be less than 10 steps.
4. Select the property that you are using in each step.

Property Choices	**Property**		**Expression** $\mathbf{(3 - 2i)(-7 + 4i)}$	
Distributive Commutative Associative Computation $i^2 = -1$		=		step 1
		=		step 2
		=		step 3
		=		step 4
		=		step 5
		=		step 6
		=		step 7
		=		step 8
		=		step 9
		=		step 10

Cluster: Use complex numbers in polynomial identities and equations.

9-11.N.CN.7 – Solve quadratic equations with real coefficients that have complex solutions.

DCAS-Like

11A

What is the solution(s) for the polynomial equation below?

$$x^2 + 4x + 53 = 0$$

A. $\{-2 + 7i, -2 - 7i\}$

B. $\{5, -9\}$

C. $\{2 + 7i, 2 - 7i\}$

D. $\{-2 + \sqrt{53i}, -2 - \sqrt{53i}\}$

Next-Generation

11B

Given the quadratic equation:

$$x^2 + 10x + 29 = 0$$

a. What are the values of a, b, and c?

b. What are the solutions to this equation? Show the process you used to solve this quadratic equation.

Answer Key and Item Rubrics

The Real Number System (N.RN)

DCAS-Equivalent Answer	**Next-Generation Solution**
1A: B (9-11.N.RN.1)	**1B:** a. Yes b. Yes c. No d. No e. Yes
2A: B (9-11.N.RN.2)	**2B:** **2 points:** TTFTF The student has a solid understanding of how to rewrite expressions involving radical and rational exponents to determine equivalent forms. **1 point:** TTFTT, TTFFF, TTTTF, TTFFT, TTTFF The student only has a basic understanding of how to rewrite expressions involving radical and rational exponents. The student can evaluate expressions containing square roots and expressions containing integer exponents as well as some simple rational exponents, such as $\frac{1}{2}$ or $\frac{3}{2}$. The student has roots raised to integer or rational exponents. The student must answer parts a and b correctly as well as at least one of the remaining parts (exception TTTTT would suggest a guessing pattern. **0 points:** All other possibilities. The student demonstrates inconsistent understanding of how to rewrite expressions involving radical and rational exponents.

DCAS-Equivalent Answer	Next-Generation Solution
3A: C (9-11.N.RN.2)	**3B:** a. We know that $\sqrt{7}$ is irrational, so we conjecture that $4+\sqrt{7}$ is irrational as well. To prove this, suppose that $4+\sqrt{7}$ were a rational number $\frac{a}{b}$, where a and b are integers. Then we would have $4+\sqrt{7}=\frac{a}{b} \quad \Rightarrow \quad \sqrt{7}=\frac{a}{b}-4$ But then $\sqrt{7}$ would be a difference of two rational numbers, which can be seen to be rational: $\sqrt{7}=\frac{a-4b}{b}$ Since $a-4b$ and b are integers, this would bean that $\sqrt{7}$ is rational, which we know to be false. So $4+\sqrt{7}$ must be irrational. Note that we may use a similar argument to show that the sum of any rational number and any irrational number is irrational. b. We know that $\sqrt{45}=\sqrt{5\times 9}=\sqrt{5}\times\sqrt{9}=3\sqrt{9}$. So, $\frac{\sqrt{45}}{\sqrt{5}}=\frac{3\sqrt{5}}{\sqrt{5}}=3,$ which is rational. c. We conjecture that $\frac{6}{\pi}$ is irrational. To prove this, suppose that $\frac{6}{\pi}$ were a rational number $\frac{a}{b}$, where a and b are integers. Then we would have: $\frac{6}{\pi}=\frac{a}{b} \quad \Rightarrow \quad \frac{6b}{a}=\pi$ Since $6b$ and a are integers, this means that π is a rational number, which we know to be false. Therefore, $\frac{6}{\pi}$ cannot be a rational number. In fact, we may use a similar argument to show that if r is any nonzero rational number and x is any irrational number, then rx and $\frac{r}{x}$ are irrational numbers.

DCAS-Equivalent Answer	Next-Generation Solution
	d. We conjecture that $\sqrt{2}+\sqrt{3}$ is irrational. If it were rational, then its square $(\sqrt{2}+\sqrt{3})^2$ would also be rational. But we have: $(\sqrt{2}+\sqrt{3})^2 = 2 + 2 \bullet \sqrt{2} \bullet \sqrt{3} + 3 = 5 + 2\sqrt{6}$ We know that $\sqrt{6}$ is irrational, and thus $5 + 2\sqrt{6}$ is also irrational (since doubling an irrational number produces an irrational number, as does adding 5 to an irrational number). Since $(\sqrt{2}+\sqrt{3})^2$ is irrational, $\sqrt{2}+\sqrt{3}$ must be irrational as well. e. $\frac{2+\sqrt{7}}{2a+\sqrt{7a^2}} = \frac{2+\sqrt{7}}{2a+\sqrt{7} \bullet \sqrt{a^2}}$ $= \frac{2+\sqrt{7}}{2a+\sqrt{7} \bullet a}$ $= \frac{2+\sqrt{7}}{a(2+\sqrt{7})}$ $= \frac{1}{a}$ Since a is a positive integer, this number is rational. (Note that rewriting $\sqrt{a^2} = a$ requires knowing that $a > 0$. In general, $\sqrt{a^2} = \|a\|$.) f. The given number may be irrational; part c. gives an example of a situation in which the sum of two irrational numbers is irrational. However, $x + y$ could be a rational number. Suppose that $x = \pi$ and $y = -\pi$. We know that x is irrational, and y is also irrational since the opposite of an irrational number is irrational. But $x + y$ is zero, which is clearly rational. Therefore, the sum of two irrational numbers can be rational or irrational.

DCAS-Equivalent Answer	Next-Generation Solution
4A: A (9-12.N.RN.3)	**4B:** *Part A:* C *Part B:* Given: l is rational; w is rational. Prove: $l \times w$ is rational. Proof: Since l is rational, by defition of rational number, l can be written in the form $\frac{a}{b}$, where a and b are both integers and b is nonzero. Similarly, since w is rational, by the definition of rational number, w can be written in the form $\frac{c}{d}$, where c and d are both integers and d is nonzero. Then, $l \times w = \frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$. Since the set of integers is closed under the operation of multiplication, both ac and bd are integers. Thus, $l \times w$ is the ratio of two integers. So by the definition of rational number, $l \times w$ is rational. ***Scoring Rubric:*** *Part A:* 1 point for selecting the correct response of C; 0 points for selecting any response other than C a. The student thinks that since the length is a whole number, so is the width. b. The student confuses what type of factors produce a product that is irrational. c. Key d. The student does not have a clear understanding of what type of factors produce a product that is irrational.

DCAS-Equivalent Answer	Next-Generation Solution
	Part B: Responses to Part B of this item will receive 0-2 points, based on the following: **2 points:** The student has a solid understanding of how to clearly and precisely construct a viable argument to support their own reasoning for proving that the product of two rational numbers is a rational number. The student clearly communicates the given information and what is to be proved. The student clearly constructs a logical sequence of steps, with reasons, to prove that the area *A* is rational. **1 point:** The student has some understanding of how to clearly and precisely construct a viable argument to support their own reasoning for proving that the product of two rational numbers is a rational number. The student communicates the given information and what is to be proved, but demonstrates some flawed or incomplete reasoning when constructing a logical sequence of steps, with reasons, to prove that the area *A* is a rational number. **0 point**: The student demonstrates inconsistent understanding of how to clearly and precisely construct a viable argument to support their own reasoning for proving that the product of two rational numbers is a rational number. The student does not clearly communicate or fails to communicate the given information or what is to be proved, and demonstrates greatly flawed or incomplete reasoning when trying to construct a logical sequence of steps, with reasons, to prove that the area *A* is a rational number.
5A: A (9-11.N.RN.3)	**5B** Solution to Item c.: 1. Always true. 2. Always true. 3. Only sometimes true—e.g., additive inverses like $\sqrt{2}$ and $-\sqrt{2}$ will cancel to 0. 4. Always true. 5. Not true—but almost! This holds except when the rational number is 0. 6. Only sometimes true—e.g., multiplicative inverses like $\sqrt{2}$ and $\frac{1}{\sqrt{2}}$ will cancel to 1.

Quantities (N.Q)

<table>
<tr><th>DCAS-Equivalent Answer</th><th>Next-Generation Solution</th></tr>
<tr><td>6A: A
(9-11.N.Q.1)</td><td>6B:
Correct answer to this item will receive 1 point.
Answer: $1\frac{1}{8}$ or 1.125 inches</td></tr>
<tr><td>7A: C
(9-11.N.Q.2)</td><td>7B:
<table>
<tr><th></th><th></th><th></th></tr>
<tr><td>a. Gives correct answer: 302,400
Shows correct work such as: $\frac{1\times7\times24\times60\times60}{2}$
Partial credit – for partially correct work subtract one point for each error.</td><td>2
3
(2)
(1)</td><td>5</td></tr>
<tr><td>b. Gives correct answer: 2734 liters—accept correct answer in milliliters (Accept answers between 2700 and 2750
Shows correct work such as:
Answer to 1 × 52
Answer to question 1 ÷ 575
Shows ÷ 10
OR
May show 86400 seconds per day × 365 days
$31536000\ seconds =\div 2$</td><td>2 ft
1 ft
1 ft
1 ft</td><td>5</td></tr>
<tr><td><i>Total Points</i></td><td></td><td><i>10</i></td></tr>
</table>
</td></tr>
</table>

<table>
<tr><th>DCAS-Equivalent Answer</th><th>Next-Generation Solution</th></tr>
<tr><td>8A: A
(9-11.N.Q.3)</td><td>

8B

a. All of the bottles have the same active ingredient and all can be diluted down to a 1% solution, so all that matters in determining value is the cost per fl oz of active ingredient. We estimate this in the following table:

<table>
<tr><th></th><th>Concentration</th><th>Price of Bottle</th><th>Cost per Ounce</th></tr>
<tr><td>A</td><td>$1.04\% \times 64 \approx 0.64 \text{ fl oz}$</td><td>$\$12.99 \approx \$13$</td><td>$\frac{13}{0.64} \approx \$20$ per fl oz</td></tr>
<tr><td>B</td><td>$18.00\% \times 32 \approx 6 \text{ fl oz}$</td><td>$\$22.99 \approx \$23$</td><td>$\frac{23}{6} \approx \$4$ per fl oz</td></tr>
<tr><td>C</td><td>$41.00\% \times 32 \approx 13 \text{ fl oz}$</td><td>$\$39.99 \approx \$40$</td><td>$\frac{40}{13} \approx \$3$ per fl oz</td></tr>
<tr><td>D</td><td>$1.04\% \times 24 \approx 0.24 \text{ fl oz}$</td><td>$\$5.99 \approx \$6$</td><td>$\frac{6}{0.24} \approx \$24$ per fl oz</td></tr>
</table>

If we assume that receiving more active ingredient per dollar is a better buy than less active ingredient per dollar, the ranking in order of best-to-worst buy is C, B, A, D.

b. The A bottles have about 0.64 fl oz of active ingredient per bottle, so to get 14 fl oz, we need $\frac{14 \text{ fl oz}}{0.64 \text{ fl oz/bottle}} \approx 22$ bottles. Purchasing 22 A bottles at about \$13 each will cost about \$286.

The B bottles have a little less than 6 fl oz of active ingredient per bottle, so to get 14 fl oz, we need 3 bottles. Purchasing 3 B bottles at about \$23 each will cost about \$69.

The C bottles have a little more than 13 fl oz of active ingredient per bottle, so we need 2 bottles. Purchasing 2 C bottles at about \$20 each will cost about \$80.

The D bottles have only 0.24 fl oz of active ingredient per bottle, so to get 14 fl oz, we need $\frac{14 \text{ fl oz}}{0.24 \text{ fl oz/bottle}} \approx 58$ bottles. Purchasing 58 D bottles at about \$6 each will cost about \$348.

Thus, although the C bottle is the cheapest when measured in dollars/fl oz, the B bottles are the best deal for this job because there is too much unused when you buy C bottles.

</td></tr>
</table>

The Complex Number System (N.CN)

<table>
<tr><th>DCAS-Equivalent Answer</th><th>Next-Generation Solution</th></tr>
<tr><td>9A: C
(9-11.N.CN.1)</td><td>9B:
a. True
b. False
c. False
d. True</td></tr>
<tr><td>10A: D
(9-11.N.CN.2)</td><td>10B:
<table>
<tr><th>Property Choices</th><th>Property</th><th></th><th>Expression
$(\mathbf{3-2}\boldsymbol{i})(\mathbf{-7+4}\boldsymbol{i})$</th><th></th></tr>
<tr><td rowspan="10">Distributive
Commutative
Associative
Computation
$i^2 = -1$</td><td>Distributive</td><td>=</td><td>$-21 + 12i + 14i - 8i$^2</td><td>step 1</td></tr>
<tr><td>Computation</td><td>=</td><td>$-21 + 26i - 8i$^2</td><td>step 2</td></tr>
<tr><td>$i^2 = -1$</td><td>=</td><td>$-21 + 26i + 8$</td><td>step 3</td></tr>
<tr><td>Computation</td><td>=</td><td>$-13 + 26i$</td><td>step 4</td></tr>
<tr><td>Commutative</td><td>=</td><td>$26i - 13$</td><td>step 5</td></tr>
<tr><td></td><td>=</td><td></td><td>step 6</td></tr>
<tr><td></td><td>=</td><td></td><td>step 7</td></tr>
<tr><td></td><td>=</td><td></td><td>step 8</td></tr>
<tr><td></td><td>=</td><td></td><td>step 9</td></tr>
<tr><td></td><td>=</td><td></td><td>step 10</td></tr>
</table>
</td></tr>
</table>

DCAS-Equivalent Answer	**Next-Generation Solution**
11A: A (9-11.N.CN.7)	**11B:** a. $a = 1$ $b = 10$ $c = 29$ b. $\frac{-10 \pm \sqrt{100-4(1)(29)}}{2a}$ $\frac{-10 \pm \sqrt{100-116}}{2}$ $\frac{-10 \pm \sqrt{-16}}{2}$ $\frac{-10 \pm i\sqrt{16}}{2}$ $\frac{-10 \pm 4i}{2}$ $-5 \pm 2i$